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Abstract
This paper examines the prospective of symmetry-breaking in terms of the property of the production possibility frontier (PPF) in a neoclassical trade model where comparative advantage depends on a domestic policy and countries have no ex-ante differences. In this model, we consider a normal-form policy game in which each country chooses the level of the domestic policy to maximize its social welfare, taking the other country's policy level as given. We show that countries necessarily choose different policy levels—that is, symmetry-breaking always occurs—after trade opens only if the PPF is not concave to the origin, and that symmetry-breaking is the only possible equilibrium outcome if the PPF is sufficiently convex to the origin in the neighborhood of the autarkic equilibrium production point. This sufficient condition for symmetry-breaking is also shown to be equivalent to that of Chatterjee (*Journal of International Economics*, 109: 102–115, 2017).

Key words: Government; Trade; Comparative advantage; Nash equilibrium; Symmetry-breaking
JEL classification: C72; F11; H50

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1. Introduction

Symmetry-breaking is a key concept for understanding how the diversity and variations across space, time, and groups emerges in economic systems. It implies the creation of asymmetric outcomes in a symmetric environment (Matsuyama, 2008) and is often modeled as an asymmetric (pure-strategy) Nash equilibrium outcome in a symmetric normal-form game (e.g., Amir et al., 2010; Chatterjee, 2017; Suga et al., 2024). In general, symmetric Nash equilibria can exist in such a symmetric game. However, if they do not exist, symmetry-breaking is the only possible equilibrium outcome. This study investigates when no symmetric Nash equilibrium exists—that is, symmetry-breaking is the inevitable outcome—in a two-country trade model with government-guided comparative advantage.

An important contribution from the same perspective as this study is Chatterjee (2017), which considers a neoclassical trade model where ex-ante identical countries choose a domestic policy that affects their comparative advantage. In this setting, the pair of optimal policies for two countries in autarky (henceforth, the autarkic policy pair) is the only candidate for a symmetric Nash equilibrium. Therefore, to ensure that symmetry-breaking is the only possible equilibrium outcome, it is sufficient to exclude the autarkic policy pair as a candidate for a Nash equilibrium. However, in her neoclassical model, each country's payoff satisfies the first order condition for local optimization in its own policy at the autarkic policy pair. Thus, she examines the condition under which each country's payoff is minimized in its own policy at the autarkic policy pair, and clarifies that the sufficient convexity of each country's aggregate income with respect to its own policy around the autarkic policy pair yields the local minimization of payoff, leading to symmetry-breaking.

This study focuses on the property of the production possibility frontier (PPF) to identify under which condition symmetry-breaking arises. In our model, which has the same production structure as in Chatterjee (2010), the PPF is given by the upper envelope of a family of restricted PPFs, each of which is defined as an upper boundary of the set of production vectors that are feasible for a given level of domestic policy. As shown in the literature on trade and productive public goods (e.g., Manning and McMillan, 1979; Clarida and Findley, 1992; Suga and Tawada, 2007; Tawada et al., 2022), such a production structure does not always yield a convex production possibility set that is familiar in the traditional, neoclassical trade theory. Can symmetry-breaking be the only possible equilibrium outcome under the convex production possibility set? What property of the PPF ensures that symmetry-breaking occurs? How is the shape of the PPF related to the sufficient condition for symmetry-breaking in Chatterjee (2017)? These are the questions with which this study is concerned.

Our main findings are as follows. If the PPF is concave to the origin, then symmetry-breaking may not arise (because a symmetric Nash equilibrium possibly exists). Symmetry-breaking is the only possible equilibrium outcome if the PPF is sufficiently convex to the origin in the neighborhood of the production point that is realized in the autarkic equilibrium with the optimal domestic policy. In addition, this sufficient condition is shown to be equivalent to that of Chatterjee (2017).

The rest of the paper is organized as follows. Sections 2 and 3 present the model and the autarkic equilibrium, respectively. Section 4 presents the trading equilibrium and the main findings. Section 4 concludes.

2. Model

We consider a two-country, two-good neoclassical model with homothetic preferences in which a domestic policy affects the determinants of comparative advantage such as relative factor endowment and relative marginal factor productivity. Here we describe the basic structure of the economy, some useful and important properties on the production side, and the production equilibrium.

2.1. Production technology

The production possibility frontier (PPF) with a given level of domestic policy (henceforth, called the restricted PPF) is expressed as

$$Q_2 = \Gamma(Q_1, R),$$

where R signifies the level of a country's domestic policy. We assume that Γ satisfies the following standard neoclassical properties:

$$\Gamma_1 < 0;\ \Gamma_{11} < 0,$$

where Γ_1 and Γ_{11} are the first and second partial derivatives of Γ with respect to Q_1, respectively. Γ is also assumed to satisfy

$$\Gamma_{1R} > 0$$

where Γ_{1R} represents the second partial derivative of Γ with respect to Q_1 and R. This implies that the slope of the PPF, or Γ_1, is increasing in R; that is, with Q_1 on the horizontal axis, an increase in R makes the PPF flatter and thus strengthens a comparative advantage in good 1.

2.2. The unrestricted production possibility frontier

The upper envelope of different restricted PPFs determined for different policy levels, or the unrestricted PPF (henceforth, referred to simply as the PPF), is given by

$$\Lambda(Q_1) \coloneqq \max_R \Gamma(Q_1, R),$$

where $\Gamma(Q_1, R)$ is assumed to be twice differentiable and have a unique maximum point on the R-axis for any given $Q_1 \geq 0$. Letting $R(Q_1)$ be the solution for the above maximization problem, we obtain the following from the first and second order conditions (i.e., $\Gamma_R\big(Q_1, R(Q_1)\big) = 0$ and $\Gamma_{RR}\big(Q_1, R(Q_1)\big) < 0$) and the envelope theorem:

$$R'(Q_1) = -\frac{\Gamma_{1R}\big(Q_1, R(Q_1)\big)}{\Gamma_{RR}\big(Q_1, R(Q_1)\big)} > 0;$$

$$\Lambda'(Q_1) = \Gamma_1\big(Q_1, R(Q_1)\big) < 0;$$

$$\Lambda''(Q_1) = \Gamma_{11}\big(Q_1, R(Q_1)\big) - \frac{\big[\Gamma_{1R}\big(Q_1, R(Q_1)\big)\big]^2}{\Gamma_{RR}\big(Q_1, R(Q_1)\big)}. \tag{1}$$

2.3. Production equilibrium

The production equilibrium under perfect competition, for a given level of domestic policy and given prices of two goods, leads to

$$p = -\Gamma_1(Q_1, R), \tag{2}$$

where p is the relative price of good 1 to good 2. Letting $Q_i(p,R)$ be the output of good i in the production equilibrium, we can use (2) to obtain the following:

$$\frac{\partial Q_1(p,R)}{\partial p} = -\frac{1}{\Gamma_{11}(Q_1(p,R),R)} > 0; \tag{3}$$

$$\frac{\partial Q_1(p,R)}{\partial R} = -\frac{\Gamma_{1R}(Q_1(p,R),R)}{\Gamma_{11}(Q_1(p,R),R)} > 0; \tag{4}$$

$$\frac{\partial Q_2(p,R)}{\partial R} = \Gamma_1(Q_1(p,R),R)\cdot\frac{\partial Q_1(p,R)}{\partial R} + \Gamma_R(Q_1(p,R),R), \tag{5}$$

where $Q_2(p,R) = \Gamma(Q_1(p,R),R)$. The aggregate income, I, is given by

$$I = G(p,R) \coloneqq pQ_1(p,R) + \Gamma(Q_1(p,R),R),$$

which, together with (2), implies that the following hold:

$$G_p = Q_1(p,R); \tag{6}$$

$$G_R = \Gamma_R(Q_1(p,R),R). \tag{7}$$

2.4. Consumption side

All consumers have the same homothetic preferences. Without loss of generality, the social indirect utility function can be written as

$$V(p,I) = I/e(p),$$

where $e(p)$ represents the unit expenditure function. The demand for good 1, $C_1(p,I)$, is obtained from Roy's identity as

$$C_1(p,I) = \phi(p)\cdot I,$$

where $\phi(p) \coloneqq e'(p)/e(p)$.

3. Autarky

Before turning to the trading equilibrium, we describe the autarkic equilibrium. For a given R, the equilibrium relative price of good 1 under autarky, $p_a(R)$, is determined to equalize relative supply and relative demand. Government determines R to maximize social utility, or

$$U(R) \coloneqq V\big(p_a(R), G(p_a(R),R)\big),$$

taking the market-clearing condition into consideration. Using (6), (7) and Roy's identity, we find that the optimal value of R satisfies

$$\begin{aligned} U'(R)/V_I &= E_1(p,R)\cdot p_a'(R) + \Gamma_R(Q_1(p,R),R) \\ &= \Gamma_R(Q_1(p,R),R) = 0, \end{aligned} \tag{8}$$

where $p = p_a(R)$ and $E_1(p,R)$ is defined as

$$E_1(p,R) \coloneqq Q_1(p,R) - C_1\big(p, G(p,R)\big), \tag{9}$$

and equals zero in the autarkic equilibrium. In what follows, we let R_a denote the optimal value of R under autarky and assume it is a unique, interior solution. (8) implies that, as shown in Figure 1, Home's autarkic equilibrium production point when $R = R_a$ is given by a production point at which the restricted PPF when $R = R_a$ is tangent to the (unrestricted) PPF; that is,

$$R_a = R\big(Q_1(p,R_a)\big), \tag{10}$$

where $p = p_a(R_a)$.

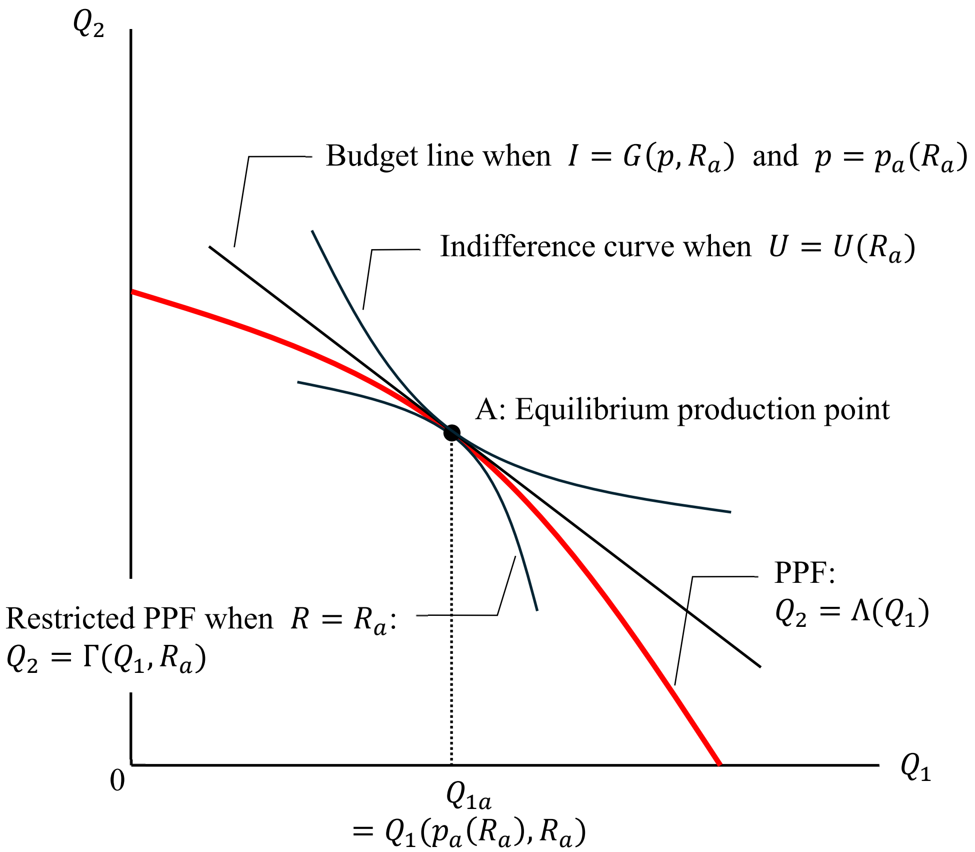

Figure 1: The autarkic equilibrium under the optimal policy level

The second derivative of social utility with respect to R when evaluated at $R = R_a$ is

$$\frac{U''(R_a)}{V_I} = \Gamma_{1R}(Q_1(p, R_a), R_a)\left[\frac{\partial Q_1(p, R_a)}{\partial p} \cdot p_a'(R_a) + \frac{\partial Q_1(p, R_a)}{\partial R}\right] + \Gamma_{RR}(Q_1(p, R_a), R_a),$$

which can be rewritten using (3) and (4) as

$$\frac{U''(R_a)}{V_I} = \frac{\partial Q_1(p, R_a)}{\partial R} \cdot p_a'(R_a) + \Gamma_{RR}(Q_1(p, R_a), R_a) - \frac{[\Gamma_{1R}(Q_1(p, R_a), R_a)]^2}{\Gamma_{11}(Q_1(p, R_a), R_a)},$$

where $p = p_a(R_a)$. Letting Q_{1a} be the value of $Q_1(p, R)$ when $p = p_a(R_a)$ and $R = R_a$, we find from (1) and (10) that the above second derivative of social utility can be expressed as

$$\frac{U''(R_a)}{V_I} = \frac{\partial Q_1(p, R_a)}{\partial R} \cdot p_a'(R_a) + \frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a}), \tag{11}$$

where $p = p_a(R_a)$.

The second order condition for utility maximization requires (11) to be negative. Evaluating (4) and (5) at $R = R_a$, we obtain the following:

$$\frac{\partial Q_1(p, R_a)}{\partial R} > 0; \ \frac{\partial Q_2(p, R_a)}{\partial R} = \Gamma_1(Q_1(p, R_a), R_a) \cdot \frac{\partial Q_1(p, R_a)}{\partial R} < 0,$$

where $p = p_a(R_a)$. These results imply that a small increase in R from R_a raises the relative supply of good 1 for a given p and thus leads to a small fall in p from $p_a(R_a)$; that is, $p_a'(R_a) < 0$. Thus, the first term on the right-hand side (RHS) of (11) is negative. However, we cannot determine the sign of second term because, while $\Gamma_{RR}(\cdot)$ and $\Gamma_{11}(\cdot)$ are both negative, the sign of $\Lambda''(Q_{1a})$ may be positive. Therefore, we assume that (11) is negative.

4. International trade

We are ready to look at the trading equilibrium. Let us consider the economy consisting of two countries, Home and Foreign, which are identical in all respects except the level of domestic policy (Superscript * is used to distinguish Foreign's variables from Home's). For given R and R^*, the equilibrium relative price of good 1 under trade, $p_t(R, R^*)$, is determined to equalize the world relative supply and the world relative demand. The symmetric structure of the two countries implies that for all R, we obtain the following:

$$p_t(R, R) = p_a(R);$$
$$\frac{\partial p_t(R, R)}{\partial R} = \frac{\partial p_t(R, R)}{\partial R^*} = \frac{p_a'(R)}{2}. \tag{12}$$

Both countries' governments play the normal-form game where each country determines its own policy level to maximize its social utility, taking the other country's policy level as given. Home's social utility under trade, or Home's payoff, is denoted as

$$U(R, R^*) \coloneqq V\big(p_t(R, R^*), G(p_t(R, R^*), R)\big).$$

Foreign's payoff is given by the same expression as Home's with the roles of R and R^* reversed. Following Chatterjee (2017), we focus on the pure-strategy Nash equilibrium (PSNE) and assume that the best responses of both countries are decreasing to ensure the existence of PSNE.[1]

In general, as shown in Chatterjee (2017), the pair of autarkic optimal policy levels of both countries, (R_a, R_a), can be a PSNE in the current model.[2] This possibility is ruled out—that is, symmetry-breaking is the only possible outcome—if each country's payoff is locally minimized in its own policy level at (R_a, R_a). In the following analysis, we will characterize this sufficient condition in terms of the shape of the PPF.

Focusing on the case of Home's payoff, let us consider how a small change in a country's policy level affects its own payoff. Suppose that both countries' initial policy levels are at the autarkic optimal level, or $R = R^* = R_a$. Then, using (6), (7), (9) and Roy's identity, we find that Home's payoff has a local optimum at $R = R_a$ when R is varied with R^* fixed at R_a:

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[1] This monotonicity of best responses for both countries is the property of a standard submodular game that ensures the existence of PSNEs (see Chapter 2 in Vives, 1999).

[2] Note that the uniqueness of the autarkic optimal policy level is necessary for (R_a, R_a) being a PSNE. To confirm this, suppose that R_{a1} and R_{a2} are different autarkic optimal policy levels. In this case, the following holds for Home's payoff:

$$U(R_{a1}, R_{a1}) = U(R_{a2}, R_{a2}) \leq U(R_{a2}, R_{a1}).$$

where the equality sign follows from that R_{ai} $(i = 1, 2)$ is the optimal policy level under autarky, and the inequality sign from the gains from trade that arises when only Foreign changes its policy level from R_{a2} to R_{a1}, with Home's policy level held at R_{a2}. With strict concave restricted PPFs for any given policy levels, the above equation holds with a strict inequality sign. In this case, (R_{a1}, R_{a1}) is not a PSNE because Home can increase its payoff by changing its policy level from R_{a1} to R_{a2}. A similar argument applies to (R_{a2}, R_{a2}).

$$\frac{1}{V_I}\cdot\frac{\partial U(R_a,R_a)}{\partial R}=E_1(p,R_a)\cdot\frac{\partial p_t(R_a,R_a)}{\partial R}+\Gamma_R(Q_1(p,R_a),R_a)=0, \tag{13}$$

where $p=p_t(R_a,R_a)=p_a(R_a)$. From (1), (3), (4) and (10) the second partial derivative of Home's payoff with respect to R when evaluated at $(R,R^*)=(R_a,R_a)$ is obtained as[3]

$$\frac{1}{V_I}\cdot\frac{\partial^2 U(R_a,R_a)}{\partial R^2}=\left[\frac{\partial Q_1(p,R_a)}{\partial p}-\frac{e''(p)I}{e(p)}\right]\left[\frac{p_a'(R_a)}{2}\right]^2$$
$$+\frac{\partial Q_1(p,R_a)}{\partial R}\cdot p_a'(R_a)+\frac{\Gamma_{RR}(Q_1(p,R_a),R_a)}{\Gamma_{11}(Q_1(p,R_a),R_a)}\cdot\Lambda''(Q_{1a}), \tag{14}$$

where $p=p_t(R_a,R_a)$ and $I=G(p,R_a)$, and Q_{1a} stands for the value of $Q_1(p,R)$ when $p=p_t(R_a,R_a)$ and $R=R_a$ This second partial derivative can have either sign.

The autarkic optimal strategy, R_a, provides a locally maximizing point of $U(R,R_a)$ if the PPF, $\Lambda(Q_1)$, is concave in the neighborhood of Q_{1a}, as shown in Figure 2. This is because, if $\Lambda''(Q_1)\leq 0$ holds around Q_{1a}, then (14) is negative. As mentioned, the first order condition (always) holds for (R_a,R_a) (see (13)), so the negative sign of (14) implies that Home's payoff is locally maximized in its own policy level at (R_a,R_a).

Lemma 1. *Home's payoff,* $U(R,R^*)$, *is locally maximized with respect to* R *at* (R_a,R_a) *if* $\Lambda''(Q_1)\leq 0$ *holds in the neighborhood of* Q_{1a}.

Proof. It is sufficient to show that the sum of the first and second terms on the RHS of (14) is negative. Note that differentiating $E_1(p_a(R),R)$ with respect to R and evaluating it at $R=R_a$, we obtain the following:

$$\frac{dE_1(p,R_a)}{dR}=\left[\frac{\partial Q_1(p,R_a)}{\partial p}-\frac{e''(p)I}{e(p)}\right]p_a'(R_a)+\frac{\partial Q_1(p,R_a)}{\partial R}=0, \tag{15}$$

where $I=G(p,R_a)$ and $p=p_a(R_a)$. $E_1(p_a(R),R)$ is the difference between equilibrium output and consumption of good 1 under autarky and therefore equals zero for all R. Thus, $dE_1(p,R_a)/dR=0$. (15) immediately implies that the sum of the first and second terms on the RHS of (14) becomes negative:

$$\left[\frac{\partial Q_1(p,R_a)}{\partial p}-\frac{e''(p)I}{e(p)}\right]\left[\frac{p_a'(R_a)}{2}\right]^2+\frac{\partial Q_1(p,R_a)}{\partial R}\cdot p_a'(R_a)$$
$$=\left\{\left[\frac{\partial Q_1(p,R_a)}{\partial p}-\frac{e''(p)I}{e(p)}\right]p_a'(R_a)+\frac{\partial Q_1(p,R_a)}{\partial R}+3\frac{\partial Q_1(p,R_a)}{\partial R}\right\}\frac{p_a'(R_a)}{4}$$
$$=\frac{\partial Q_1(p,R_a)}{\partial R}\cdot\frac{3p_a'(R_a)}{4}<0.$$

Q.E.D.

[3] See Appendix for details of the derivation of (14).

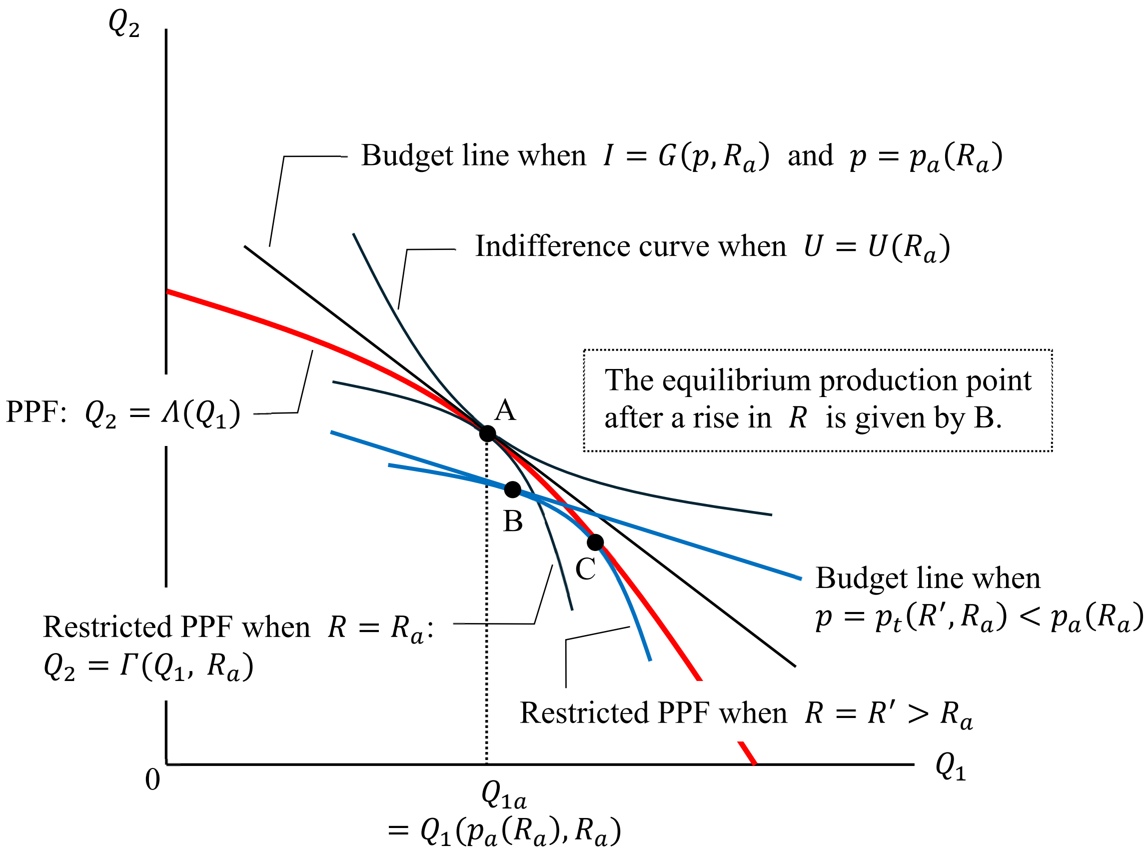

Figure 2: The effect of an increase in R when $\Lambda''(Q_1) \leq 0$ holds around Q_{1a}

From the proof of Lemma 1, we find that the strict convexity of $\Lambda(Q_1)$ in the neighborhood of Q_{1a}, or $\Lambda''(Q_{1a}) > 0$, is necessary for $U(R, R_a)$ not to be locally maximized at $R = R_a$, as shown in Figure 3. Furthermore, if $\Lambda''(Q_{1a})$ is sufficiently large, (R_a, R_a) is excluded as a candidate for the PSNE.[4]

Proposition 1. *Suppose that (11) is negative, that is, the second order condition for the maximization problem under autarky holds. Then symmetry-breaking is the only possible PSNE outcome if the convexity of* $\Lambda(Q_1)$ *in the neighborhood of* Q_{1a} *is strong enough to satisfy*

$$\frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a})$$

$$> -\left[\frac{\partial Q_1(p, R_a)}{\partial p} - \frac{e''(p)I}{e(p)}\right]\left[\frac{p_a'(R_a)}{2}\right]^2 - \frac{\partial Q_1(p, R_a)}{\partial R} \cdot p_a'(R_a), \qquad (16)$$

where $I = G(p, R_a)$ *and* $p = p_t(R_a, R_a) = p_a(R_a)$.

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[4] Note that $\Lambda''(Q_{1a})$ has the upper bound. For $p = p_a(R_a)$, it must satisfy

$$-\frac{\partial Q_1(p, R_a)}{\partial R} \cdot p_a'(R_a) > \frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a}),$$

which is equivalent to (11) being negative, i.e., the second order condition for maximization under autarky.

Proof. (16) implies that (14) is positive, so Home's payoff is locally minimized at $R = R_a$ when making R vary with R^* fixed at R_a. Thus, Home can be better off by slightly deviating from $R = R_a$ when $R^* = R_a$; that is, (R_a, R_a) is not a PSNE. **Q.E.D.**

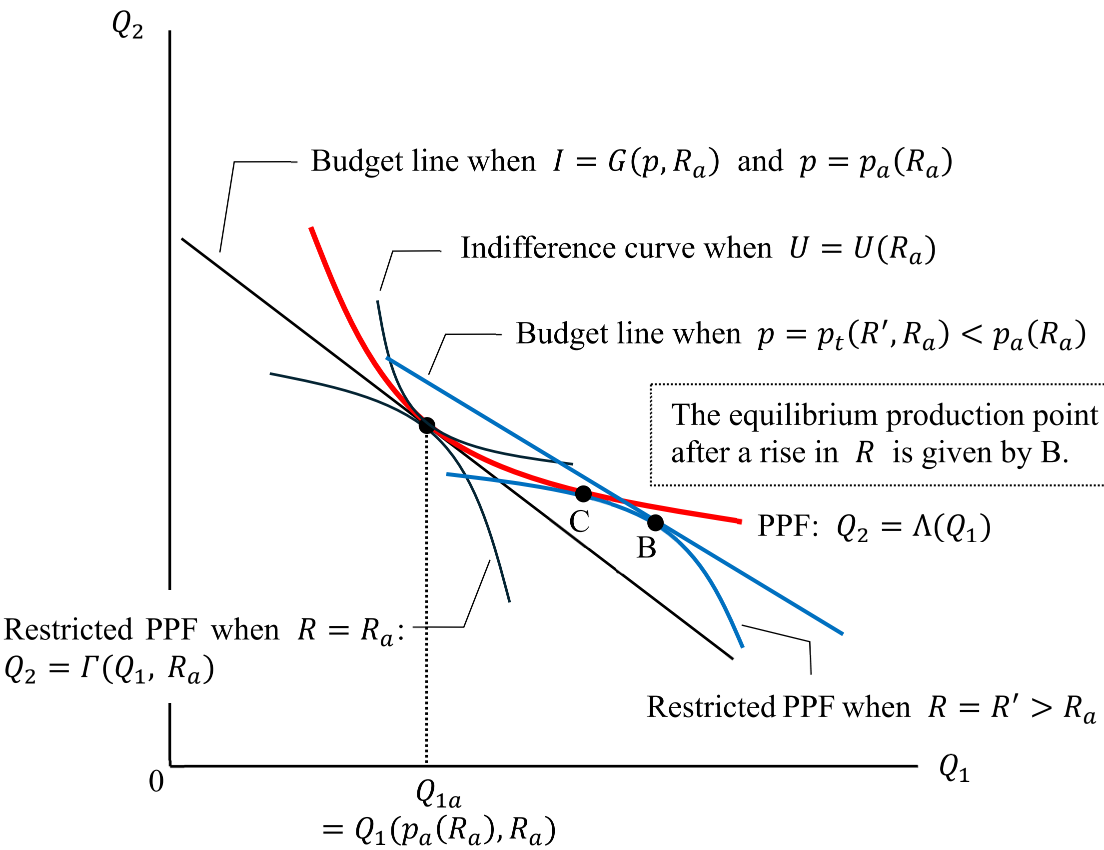

Figure 3: The effect of an increase in R when (16) holds

The sufficient convexity of the PPF at the neighborhood of Q_{1a}, or (16), is equivalent to Chatterjee's (2017) sufficient condition for symmetry-breaking, or to that the aggregate income, $G(p_t(R, R^*), R)$, is sufficiently convex in its own policy level around (R_a, R_a). This is because the LHS of (16) is equal to $G_{RR}(p_t(R_a, R_a^*), R_a)$. This equivalence can be shown as follows:

$$\begin{aligned} G_{RR}(p, R_a) &= \Gamma_{R1}(Q_1(p, R_a), R_a)\frac{\partial Q_1(p, R_a)}{\partial R} + \Gamma_{RR}(Q_1(p, R_a), R_a) \\ &= \Gamma_{RR}(Q_1(p, R_a), R_a) - \frac{[\Gamma_{R1}(Q_1(p, R_a), R_a)]^2}{\Gamma_{11}(Q_1(p, R_a), R_a)} \\ &= \frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)}\left\{\Gamma_{11}(Q_1(p, R_a), R_a) - \frac{[\Gamma_{R1}(Q_1(p, R_a), R_a)]^2}{\Gamma_{RR}(Q_1(p, R_a), R_a)}\right\} \\ &= \frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a}), \end{aligned}$$

where $p = p_t(R_a, R_a)$. The second equality follows from (4) and the last equality from (1).

5. Conclusion

This paper has analyzed when symmetry-breaking arises as the only possible PSNE outcome under policy-guided comparative advantage in a neoclassical trade model with ex-ante identical countries. It finds that symmetry-breaking is inevitable only if the PPF is not concave to the origin. That is, the traditional concave PPF makes it impossible to explain as an inevitable result of trade liberalization the observed differences between similar countries under trade. Our analysis discloses that symmetry-breaking is the only possible PSNE outcome if the PPF is sufficiently convex to the origin in the neighborhood of the autarkic equilibrium production point. We also find that this sufficient condition for symmetry-breaking is equivalent to that of Chatterjee (2017), or that each country's aggregate income is sufficiently convex with respect to its own policy level around the pair of autarkic optimal policy levels of both countries.

Appendix: Derivation of (14)

Differentiating twice Home's payoff with respect to R and evaluating this second partial derivative at $(R, R^*) = (R_a, R_a)$, we obtain

$$\frac{1}{V_I} \cdot \frac{\partial^2 U(R_a, R_a)}{\partial R^2} = \frac{dE_1(p, R_a)}{dR} \cdot \frac{\partial p_t(R_a, R_a)}{\partial R}$$
$$+\Gamma_{1R}(Q_1(p, R_a), R_a)\left[\frac{\partial Q_1(p, R_a)}{\partial p} \cdot \frac{\partial p_t(R_a, R_a)}{\partial R} + \frac{\partial Q_1(p, R_a)}{\partial R}\right].$$
$$+\Gamma_{RR}(Q_1(p, R_a), R_a),$$

This can be rewritten using (3) and (4) as

$$\frac{1}{V_I} \cdot \frac{\partial^2 U(R_a, R_a)}{\partial R^2} = \left[\frac{dE_1(p, R_a)}{dR} + \frac{\partial Q_1(p, R_a)}{\partial R}\right] \frac{\partial p_t(R_a, R_a)}{\partial R}$$
$$+\Gamma_{RR}(Q_1(p, R_a), R_a) - \frac{[\Gamma_{1R}(Q_1(p, R_a), R_a)]^2}{\Gamma_{11}(Q_1(p, R_a), R_a)}.$$

where $p = p_t(R_a, R_a) = p_a(R_a)$. Letting Q_{1a} be the value of $Q_1(p, R)$ when $p = p_t(R_a, R_a)$ and $R = R_a$, we find from (1) and (10) that the above second partial derivative of Home's payoff can be expressed as

$$\frac{1}{V_I} \cdot \frac{\partial^2 U(R_a, R_a)}{\partial R^2} = \left[\frac{dE_1(p, R_a)}{dR} + \frac{\partial Q_1(p, R_a)}{\partial R}\right] \frac{\partial p_t(R_a, R_a)}{\partial R}$$
$$+\frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a}) \qquad \text{(A1)}$$

Totally differentiating $E_1(p, R)$ and evaluating it at $(R, R^*) = (R_a, R_a)$ yields

$$\frac{dE_1(p, R_a)}{dR} = \left[\frac{\partial Q_1(p, R_a)}{\partial p} - \frac{e''(p)I}{e(p)}\right] \frac{dp}{dR} + \frac{\partial Q_1(p, R_a)}{\partial R}, \qquad \text{(A2)}$$

where $I = G(p, R_a)$ and $p = p_t(R_a, R_a)$. Inserting (A2) into (A1) and taking $dp/dR = \partial p_t(R_a, R_a)/\partial R$ into account, we obtain the following:

$$\frac{1}{V_I} \cdot \frac{\partial^2 U(R_a, R_a)}{\partial R^2} = \left[\frac{\partial Q_1(p, R_a)}{\partial p} - \frac{e''(p)I}{e(p)}\right] \left[\frac{\partial p_t(R_a, R_a)}{\partial R}\right]^2$$
$$+2\frac{\partial Q_1(p, R_a)}{\partial R} \cdot \frac{\partial p_t(R_a, R_a)}{\partial R} + \frac{\Gamma_{RR}(Q_1(p, R_a), R_a)}{\Gamma_{11}(Q_1(p, R_a), R_a)} \cdot \Lambda''(Q_{1a}).$$

Moreover, using (12) in the above, we obtain (14).

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