Multiclass traffic assignment model considering heterogeneous stochastic headways of autonomous vehicles and human-driven vehicles

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Abstract This study proposed a stochastic link capacity in mixed flows, considering heterogeneous stochastic headways for autonomous vehicles (AVs) and human-driven vehicles (HVs), respectively. Using this link traffic capacity, a multi-class traffic assignment problem considering mixed flows is proposed. Though previous study assumed an approximation of random variable computation to derive mixed link capacity, by assuming lognormally distributed stochastic headway, we showed the mixed link capacity model that ensure the link capacity analytically following lognormal distribution without any approximate computation of stochastic variables. Furthermore, the analytical relationship between the mean and standard deviation of the stochastic link capacity and the market penetration rate of AVs in a link is derived. We also showed the conditions for the mean and variance of the link travel time to be monotonically increasing with the link flow. The proposed traffic assignment model assumes that AVs and HVs follow UE and SUE, respectively. Numerical calculations were performed to validate the proposed model using a test network.

Keywords autonomous vehicle, stochastic mixed traffic, travel time reliability, multiclass traffic assignment

1 Introduction

In recent years, automatic driving technology has been developing. The various benefits from the adoption of the autonomous vehicles (AVs) have been discussed in the literature (e.g., Fagnant and Kockelman [1]). Especially, it is expected that the spread of autonomous vehicles (AVs) mitigates congestion and reduces travel time because the critical headway (CH) of AVs can be smaller than that of human-driven vehicles (HVs). In the large penetration rate of AVs in a link, the road space can be utilized efficiently and the link capacity of the mixed flow of AVs and HVs increase resulting in the mitigation of congestion and the reduction of travel time.

It is expected that AVs and HVs share the same road space for a long time after the first appearance of AVs. To represent the mixed flow of AVs and HVs, the link capacity under the mixed flow is studied in the static models (e.g., Levin and Boyles [2]; Seo and Asakura [3]; Liu and Song [4]; Wang et al. [5-7]; Tani et al. [8]) and the dynamic models (e.g., Levin and Boyles [9]; Ghiasi et al. [10]; Zhou and Zhu [11]; Guo et al. [12]; Chen et al. [13]). The existing studies mentioned above can be classified into two categories: those that assume deterministic traffic capacity and those that assume stochastic traffic capacity. Levin and Boyles [2] proposed a traffic capacity model based on the speed-density relationship proposed by Greenshields et al. [14], where the heterogeneous and deterministic headways of AVs and HVs are considered. van den Berg and Verhoef [15] analyzed how the dissemination of AVs might affect traffic capacity and the driver's value of time. Le Vine et al. [16] analyzed the variability of link capacity during the dissemination of AVs, focusing on the driving behavior of AVs. Wang et al. [5] proposed a link capacity model under mixed flow. The similarity of the above studies is to assume the deterministic traffic capacity. However, the stochastic nature of traffic capacity has been widely pointed out by the reliability studies of the road network (e.g., Chen et al. [17]; Uchida [18]), and exists in the mixed flow of AVs and HVs.

There exist two causes of the stochastic nature of traffic capacity in the literature: (i) stochastic order of AVs and HVs in a platoon and (ii) heterogeneity of stochastic headway between leading and following vehicle. Ghiasi et al. [10] considered the stochasticity of the traffic capacity under the mixed flow based on the first cause, the stochastic vehicle order in platoon. Ghiasi et al. formulated the stochastic traffic capacity with two different deterministic headways of AVs and HVs in the dynamic traffic flow. The stochasticity of their traffic capacity results from the uncertainty of the platooning patterns of AVs and HVs in a traffic flow. The model proposed in their study requires the platooning intensity of AVs which is given exogenously. Because their proposed model is used in the dynamic flow, their model cannot be applied to network-level static traffic assignment model. Zhou and Zhu [11] considered the second cause, the stochastic headway in the mixed flow. Zhou and Zhu proposed a dynamic capacity model for the stochastic mixed flow of AVs and HVs in a corridor network. To consider the uncertainty of travel times under the mixed flow in the static network equilibrium, Tani et al. [8] proposed a variable and stochastic link capacity under the mixed traffic. Their link capacity model is based on Wang et al. [5], and assumes the traffic capacities of pure-AV flow and pure-HV flow to compute the traffic capacity of the mixed flow. As far as the authors know, Tani et al. [8] is the only study to apply the stochastic traffic capacity under the mixed flow to a network model. However, their model was not explicitly deduced from the heterogeneous CHs of AVs and HVs.

To fill in the research gap, this study formulates the stochastic traffic capacity under the mixed flow considering the heterogeneous CHs of AVs and HVs. The traffic capacity can be applied to the network-based traffic assignment problem. The CHs of AVs and HVs are represented as random variables, respectively. The heterogeneity of mean and variance of CHs of AVs and HVs can be addressed in the proposed traffic capacity model.

The path choice principles of AVs and HVs have been also discussed in the literature addressing the traffic assignment models considering the traffic capacity under the mixed traffic flow. Bagloee et al. [19] and Zhang and Nie [20] assumed that some demands followed user equilibrium and others followed system optimum, where AVs were controlled by the central agent to decrease the total travel time. On the other hand, Chen et al. [21] and Liu and Song [4] assumed both AVs and HVs followed user equilibrium principle. Wang et al. [5, 6] assumed that AVs could acquire more accurate traffic information than HVs so that AVs and HVs follow user equilibrium principle and cross-nested logit principle, respectively. Following Wang's studies, this study focuses on the difference in the accuracy of the travel time information between AVs and HVs and assumes AVs and HVs follow user equilibrium principle and stochastic user equilibrium principle, respectively.

When the stochastic link capacity is incorporated into a traffic assignment model, the travel times of link and path become stochastic variables, where the drivers choose their paths based on the stochastic travel time. In order to address the uncertainty of the mixed traffic of AVs and HVs, the outcomes developed in the literature addressing the traffic assignment model considering the travel time reliability, of course in the pure HV network can be utilized. In the literature on the network-based model considering the travel time reliability, the stochastic traffic capacity as well as the stochastic traffic flow were addressed and treated as the source of the uncertainty of the travel time. Some studies proposed traffic assignment models that introduced stochastic traffic demand following normal distributions to express stochastic travel times (Nakayama and Takayama [22]; Clark and Watling [23]; Lam et al. [24]; Uchida [25]). If the traffic demand follows a normal distribution and the link travel time is expressed by a polynomial function of the stochastic link flow, then an approximation method is needed to calculate the mean and variance of the stochastic link travel time. Traffic flow assignment models in which traffic demand is assumed to follow a log-normal distribution were proposed (Zhou and Chen [26]; Sumalee and Xu [27]; Tani et al. [8]). Some studies addressed stochastic traffic capacity in order to express stochastic travel times (Chen et al. [17]; Tani et al. [8]). The former assumed that the link capacity follows a uniform distribution, whereas the latter assumed that the link capacity follows a log-normal distribution. Especially in Tani et al. [8], since traffic demand and traffic capacity both follow log-normal distributions, the corresponding link travel time follows a shifted log-normal distribution. The empirical studies of Uno et al. [28] and Srinivasan et al. [29] reported that link travel times in road networks can fit to a shifted log-normal distribution from the observed data of travel times. Based on their studies, it is reasonable to represent the travel time distribution as such a nonnegative and asymmetric distribution in a traffic assignment model.

Thereby, this study proposes a novel traffic assignment model incorporating the heterogeneous stochastic headways of AVs and HVs. The traffic assignment model simultaneously addresses stochastic network variables, i.e., traffic flow, travel time, traffic capacity, and the path choice behavior under the uncertain network.

The contributions of this study are twofold. First, we formulate a stochastic link capacity of the mixed flow considering the heterogeneous stochastic CHs of AVs and HVs, which was not explicitly addressed in the existing study (Tani et al. [8]). It not only improves the realism of the mixed traffic behavior but also enables the evaluation of the impact of the changes in the mean and variance of AV CH on the network performance, such as flow patterns and path travel time.

The remainder of this paper is organized as follows. Section 2 presents the formulation of the stochastic network variables, i.e., traffic flow, link capacity, and travel time, under the stochastic mixed flow of AVs and HVs. Section 3 presents a multiclass traffic assignment model under the stochastic mixed flow. Section 4 shows the results of numerical experiments carried out to demonstrate the model described in Section 3. Finally, we conclude this paper in Section 5.

2 Flows and travel time

In this section, we formulate the stochastic traffic flow, link capacity, and travel time. As assumed in the literature (Zhou and Chen [26]; Sumalee and Xu [27]; Tani et al. [8]), the traffic demand in this study follows a log-normal distribution. Following Tani et al. [8], the traffic capacity in this study follows a log-normal distribution. This assumption can be supported by the insight shown by Chen et al. [13] that a log-normal distribution is one of the better-fitting distributions of link capacity. As will be shown later, these two assumptions are consistent with the evidence shown by Uno et al. [28] and Srinivasan et al. [29] that link travel times in road networks follow a shifted log-normal distribution. Notations used in the formulation are summarized in Appendix A.

2.1 Traffic flow

The network flows in this study are represented as random variables to express uncertainties of travel times in the network. Total traffic demand Q in the network is a random variable that follows the following log-normal distribution (Tani et al. [8]):

$$Q \sim LN(\mu_Q, \sigma_Q^2) \tag{1}$$

If a random variable X follows a log-normal distribution $LN(\mu_X, \sigma_X^2)$, then $E[X]$ and $Var[X]$ are calculated as $\exp(\mu_X + \sigma_X^2/2)$ and $\exp(2\mu_X + \sigma_X^2) \cdot (\exp(\sigma_X^2) - 1)$, respectively. The total traffic demand of type-z vehicles Q_z is represented as:

$$Q_z = p_z \cdot Q \quad \forall z \in \{h, a\} \tag{2}$$

where $\sum_{z \in \{h,a\}} p_z = 1$ and $p_z \geq 0$ $(\forall z \in \{h, a\})$. p_a and p_h are the penetration rates of AVs and HVs of the total traffic demand in the entire road network, respectively. Thereby, p_a and p_h can be interpreted as the market penetration rates of AVs and HVs in the entire road network, respectively. To express OD demand, we introduce the ratio of OD demand for type z vehicles to Q_z denoted by $p_{z,w}$. Then, the traffic demand of type z vehicles between OD pair w, $Q_{z,w}$, is:

$$Q_{z,w} = p_{z,w} \cdot Q_z \quad \forall z \in \{h, a\}, \forall w \in W \tag{3}$$

where

$$\sum_{w \in W} p_{z,w} = 1 \ \left(p_{z,w} \geq 0\right) \forall z \in \{h, a\} \tag{4}$$

The flow of type z vehicles on path k between OD pair w, $F_{z,w,k}$, is:

$$F_{z,w,k} = p_{z,w,k} \cdot Q_{z,w} = p_{z,w,k} \cdot p_{z,w} \cdot p_z \cdot Q \quad \forall z \in \{h, a\}, \forall w \in W, \forall k \in K_w \tag{5}$$

where $\sum_{k \in K_w} p_{z,w,k} = 1$ $(\forall w \in W, \forall z \in \{h, a\})$ and $p_{z,w,k} \geq 0$ $(\forall k \in K_w, \forall w \in W, \forall z \in \{h, a\})$. The link flow is then expressed as the sum of all path flows that pass through the link given by:

$$\begin{aligned} V_{z,s} &= \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,s} \cdot F_{z,w,k} \\ &= \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,s} \cdot p_{z,w,k} \cdot Q_{z,w} \\ &= \hat{p}_{z,s} \cdot Q \ \forall z \in \{h, a\}, \forall s \in S \end{aligned} \tag{6}$$

where

$$\hat{p}_{z,s} = \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,s} \cdot p_z \cdot p_{z,w} \cdot p_{z,w,k} \quad \forall z \in \{h, a\}, \forall s \in S \tag{7}$$

The link flow is represented as the product of the scaler $\hat{p}_{z,s}$ and the total traffic demand Q. Thereby, the total link flow, V_s, for both types of vehicles is given as the sum of the link flows of AVs and HVs.

$$\begin{aligned} V_s = V_{h,s} + V_{h,s} &= \hat{p}_{h,s} \cdot Q + \hat{p}_{a,s} \cdot Q \\ &= \hat{p}_s \cdot Q \ \ \forall s \in S \end{aligned} \tag{8}$$

where

$$\hat{p}_s = \hat{p}_{h,s} + \hat{p}_{a,s} \ \forall s \in S \tag{9}$$

Here, $\hat{p}_{z,s}$ and $\hat{p}_s$ are the proportions of $V_{z,s}$ and V_s to total traffic demand Q. Hereafter referred to as link flow proportion.

From (8), V_s follows a log-normal distribution shown as:

$$V_s \sim LN\left(\mu_{V_s}, \sigma_{V_s}^2\right) \ \ \forall s \in S \tag{10a}$$

where

$$\mu_{V_s} = \mu_Q + \ln(\hat{p}_s) \tag{10b}$$

$$\sigma_{V_s}^2 = \sigma_Q^2 \tag{10c}$$

The traffic flow formulation shown in this section enables the analytical representation of the link flow distribution. The link flow in this study follows a log-normal distribution, and its mean and variance can be calculated using the distribution parameters, μ_Q and σ_Q^2, of the total demand. This calculation requires neither the enumeration of all paths that pass through the link nor the calculation of covariance between two path flows. This mathematical property comes from the assumption shown by (1) and contributes to reducing the calculation load in traffic assignments or network design problems. When calculating the variance of a link flow, it should be noted that if OD flows are represented by independent random variables, then all paths that pass through the link need to be enumerated in order for the path flow covariance to be calculated (Lam et al. [24]). In addition, the corresponding link flow no longer follows the same distribution as assumed in OD flows due to the covariance of two path flows. Therefore, in the literature, the distribution is approximated by a theoretical distribution, e.g., when OD flows follow independent normal distributions, link flows are assumed to follow normal distributions (Lam et al. [24]). As shown in a later section, this mathematical property also helps the link travel time distribution to be calculated correctly. Namely, the link travel time in this study follows a shifted log-normal distribution. It is also noted that when the link flow follows a normal distribution, in general, no corresponding theoretical distribution of the link travel time exists. Therefore, in the literature, the distribution is assumed to follow a theoretical distribution with statistical parameters.

2.2 Link Capacity

The link capacity can be calculated as the inverse of the corresponding CH. The CH is the minimum headway time. The headway time is defined as the time elapsed between the front of the lead vehicle passing a point on the roadway and the front of the following vehicle passing the same point. The link flow in this study follows a log-normal distribution. Therefore, the link capacity in this study also follows a log-normal distribution since the link capacity is the maximum link flow that follows a log-normal distribution. In addition, all AVs in the network can use the control system to engage in the same driving maneuvers. In contrast, since HVs are controlled by a human, their driving maneuvers vary from individual to individual. In the mixed traffic flow of AVs and HVs, the headway times vary due to HV maneuvers. To represent such variation in headway times, we introduce stochastic CH. As a result, the link capacity, i.e., the inverse of the stochastic CH, in this study is calculated as a stochastic variable, as shown below.

The CH for the type z vehicle in this study is calculated as the basic CH, h_0, times the ratio of the CH for a type z vehicle to the basic CH of R_z. We consider then a vehicle platoon that consists of AVs and HVs. The AV penetration rate in the platoon is $\omega_{a,s} \in [0,1]$. Each AV/HV in this study has a CH ratio that is randomly sampled from the log-normal distribution of $LN\left(\mu_{R_z}, \sigma_{R_z}^2\right)$ $(z \in \{h,a\})$. Now we consider the CH ratio of a vehicle that is randomly selected from the platoon. The ith sampled CH ratio for each type of vehicle $z \in \{h,a\}$ is denoted as r_z^i. The number of samplings is n_s. The geometric mean of the sequence of the sampled CH ratios, r_s, is given by:

$$r_s = \left(\hat{r}_{a,s} \cdot \hat{r}_{h,s}\right)^{\frac{1}{n_s}} = \left(\hat{r}_{a,s}\right)^{\frac{1}{n_s}} \cdot \left(\hat{r}_{h,s}\right)^{\frac{1}{n_s}} \tag{11}$$

where $\hat{r}_{a,s} = \prod_{i=1}^{n_s^a} r_a^i$, $\hat{r}_{h,s} = \prod_{j=1}^{n_s^h} r_h^j$ and $n_s = n_{a,s} + n_{h,s}$. In (11), $n_{a,s}$ and $n_{h,s}$ are the numbers of AVs and HVs sampled, respectively. Note that the geometric mean is the appropriate measure of the central tendency calculated using the sampled values from a log-normal distribution. According to (11), $\hat{r}_{a,s}$ and $\hat{r}_{h,s}$ can be regarded as random samplings from $\hat{R}_{a,s}(= R_a^{n_{a,s}}) \sim LN\left(n_{a,s} \cdot \mu_{R_a}, \left(n_{a,s}\right)^2 \cdot \sigma_{R_a}^2\right)$ and $\hat{R}_{h,s}(= R_h^{n_{h,s}}) \sim LN\left(n_{h,s} \cdot \mu_{R_h}, \left(n_{h,s}\right)^2 \cdot \sigma_{R_h}^2\right)$, respectively. $\hat{R}_{z,s}$ represents the $n_{z,s}$ order moment of R_z . If n_s is sufficiently large, then $\frac{n_{a,s}}{n_s} = \omega_{a,s}$. Therefore, r_s can be regarded as a sample generated from $R_s \sim LN\left(\mu_{R_s}, \sigma_{R_s}^2\right)$ where $\mu_{R_s} = \omega_{a,s} \cdot \mu_{R_a} + \left(1 - \omega_{a,s}\right) \cdot \mu_{R_h}$ and $\sigma_{R_s}^2 = \left(\omega_{a,s}\right)^2 \cdot \sigma_{R_a}^2 + \left(1 - \omega_{a,s}\right)^2 \cdot \sigma_{R_h}^2$. If the samplings of r_a^i and r_h^j and the calculation of r_s from the samples are repeated in sufficiently numerous times, then the stochastic ratio of CH on link s, R_s is obtained from the numerous samples of r_s, shown as:

$$R_s = R_a^{\omega_{a,s}} \cdot R_h^{1-\omega_{a,s}} \tag{12a}$$

where

$$\omega_{a,s} = \frac{V_{a,s}}{V_s} = \frac{\hat{p}_{a,s}}{\hat{p}_s} \tag{12b}$$

Therefore, the ratio of the CH for the mixed flow of AVs and HVs to the basic CH on link s, R_s, has the following log-normal distribution:

$$R_s \sim LN(\mu_{R_s}, \sigma_{R_s}^2) \tag{13}$$

Consequently, the CH of vehicles on link s is calculated as:

$$H_s = h_0 \cdot R_s \tag{14}$$

From the definition of the hourly capacity of link s, if the unit of CH is second, it is represented as:

$$C_s = \frac{3600}{H_s} \tag{15}$$

From (13)-(15), C_s has the following log-normal distribution:

$$C_s \sim LN(\mu_{C_s}, \sigma_{C_s}^2) \tag{16a}$$

where

$$\mu_{C_s} = \ln(3600) - \ln(h_0) - \mu_{R_s} \tag{16b}$$

$$\sigma_{C_s}^2 = \sigma_{R_s}^2 \tag{16c}$$

Thereby, the parameters of link capacity, C_s vary with the AV penetration rate of link s, $\omega_{a,s}$. Note that the mathematical properties of the link capacity are given in Appendix B.

2.3 Travel time

The model proposed in this paper falls within the category of static models for long-term planning at the strategic level. Therefore, both the uniqueness and the stability of a solution by the proposed model should be discussed. Although the uniqueness of the solution may not be guaranteed due to the asymmetric effect of mixed flow on the link capacity, the stability of the solution may be guaranteed under a continuously differentiable link travel time function that increases monotonically with respect to the link flow. Considering this, we use a BPR function to calculate the link travel time. The link travel time in this study is calculated by the following BPR function:

$$T_s\left(V_{h,s}, V_{a,s}\right) = t_s^0 \cdot \left(1 + \alpha_s \cdot \left(\frac{V_s}{C_s}\right)^{\beta_s}\right) \ \forall s \in S \tag{17}$$

where C_s is the link capacity shown by (16). By substituting (8) and (16) into (17), we obtain:

$$T_s(V_{a,s}, V_{h,s}) = t_s^0 + T_s' \tag{18}$$

where

$$T_s' = t_s^0 \cdot \alpha_s \cdot \left(\frac{\hat{p}_s \cdot Q \cdot H_s}{3600}\right)^{\beta_s} \ \forall s \in S \tag{19}$$

(18) is composed of a free-flow travel time term (the first term, t_s^0) and a delay time term due to congestion (the second term, T_s'). T_s' has the following log-normal distribution:

$$T_s' \sim LN(\mu_{T_s'}, \sigma_{T_s'}^2) \tag{20a}$$

where

$$\mu_{T_s'} = \ln(t_s^0) + \ln(\alpha_s) + \beta_s \cdot \{\mu_Q + \mu_{R_s} + \ln(\hat{p}_s) + \ln(h_0) - \ln(3600)\} \tag{20b}$$

$$\sigma_{T_s'}^2 = (\beta_s)^2 \cdot (\sigma_Q^2 + \sigma_{R_s}^2) \tag{20c}$$

Accordingly, the link travel time in this study follows a shifted log-normal distribution and is consistent with the evidence shown by Uno et al. [28] and Srinivasan et al. [29]. Then, the mean link travel time, the link travel time variance, and the covariance of two link travel times are respectively represented as:

$$E[T_s] = t_s^0 + E[T_s'] = t_s^0 + \exp\left(\mu_{T_s'} + \frac{1}{2} \cdot \sigma_{T_s'}^2\right) \tag{21}$$

$$Var[T_s] = Var[T_s'] \quad (22)$$
$$= \exp\left(2\mu_{T_s'} + \sigma^2_{T_s'}\right)\left(exp\left(\sigma^2_{T_s'}\right) - 1\right))$$

$$Cov[T_{s1}, T_{s2}] = Cov[T_{s1}', T_{s2}'] \quad (23a)$$
$$= \exp\left(\mu_{T_{s1}'\cdot T_{s2}'} + \frac{1}{2}\cdot\sigma^2_{T_{s1}'\cdot T_{s2}'}\right)$$
$$-\exp\left(\mu_{T_{s1}'} + \mu_{T_{s2}'} + \frac{1}{2}\cdot\left(\sigma^2_{T_{s1}'} + \sigma^2_{T_{s2}'}\right)\right)$$
$$\forall s1, s2 \in S$$

where

$$\mu_{T_{s1}'\cdot T_{s2}'} = \ln(\alpha_{s1}) + \ln(\alpha_{s2}) + \ln(t^0_{s1}) + \ln(t^0_{s2}) + \beta_{s1}\cdot\ln(\hat{p}_{s1}) + \beta_{s2}\cdot\ln(\hat{p}_{s2}) + (\beta_{s1}+\beta_{s2})\cdot\mu_Q - \beta_{s1}\cdot\mu_{C_{s1}} - \beta_{s2}\cdot\mu_{C_{s2}} \quad (23b)$$

$$\sigma^2_{T_{s1}'\cdot T_{s2}'} = (\beta_{s1}+\beta_{s2})^2\cdot\sigma^2_Q + \beta^2_{s1}\cdot\sigma^2_{C_{s1}} + \beta^2_{s2}\cdot\sigma^2_{C_{s2}} \quad (23c)$$

Figures 1 and 2 respectively show the changes in mean and standard deviation of link travel time when the link flow proportion and the penetration of AVs in the link change. The free-flow travel time of the link is set as 5 [min]. The parameters of the BPR function of the link, α_s and β_s, are set as 0.15 and 4, respectively. The mean and the coefficient of variation of the total traffic demand are respectively set as 18,000 [pcu/hour] and 0.10 ($Q\sim LN(9.79, 9.95\times10^{-3})$), where *pcu* represents a passenger car unit. We set h_0, μ_{R_h}, and $\sigma^2_{R_h}$ as 2.0 [sec], 0.14, and 0.04, respectively. The corresponding mean HV CH is calculated as 2.35 [sec]. We set μ_{R_h} and $\sigma^2_{R_a}$ as 0 and 0.03, respectively. The parameter settings of the link capacity hold the conditions shown in Appendix C, which ensure that the link travel cost in this study, $E[T_s] + \gamma\cdot\sqrt{Var[T_s]}$ $(\gamma > 0)$, increases monotonically with increases in the mean link flow. As an example, the parameters are set following the results of Appendix C for the numerical calculation. However, in practice, these parameters should be estimated from the observation of the real road network and the expected AV performance rather than freely set by the analyst. In these figures, the mean link travel time seems not to change much at a low link flow proportion. The mean link travel time is calculated by (21). At a low link flow proportion, the numerator of the second term, i.e., link flow V_s, takes a small value. On the other hand, the denominator of the second term, i.e., link capacity C_s, is calculated depending only on (the relative ratio of AVs and HVs in a total link flow) its link AV penetration rate $\omega_{a,s}$, not depending on the link flow proportion itself. Therefore, the change in mean link travel time is small at a low link flow proportion. The link travel cost corresponds to the path travel cost in this study, which is defined later in this section.

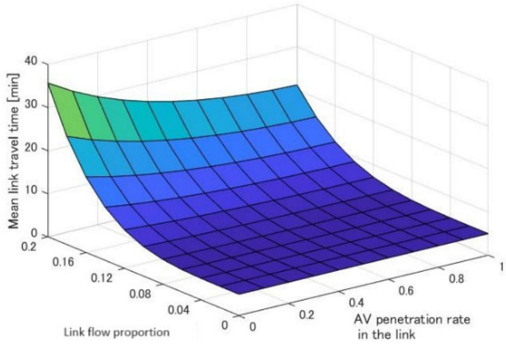

Figure 1. Mean link travel time

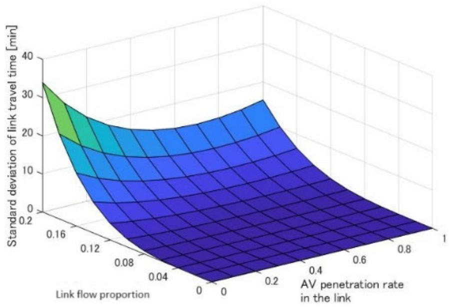

Figure 2. Standard deviation of link travel time

The path travel time is expressed as the sum of the travel times of the links that comprise the path:

$$PT_{w,k} = \sum_{s\in S} T_s\cdot\delta_{w,k,s}\ \forall w\in W, \forall k\in K_w \quad (24)$$

The mean and variance of path travel time are respectively represented as:

$$E[PT_{w,k}] = \sum_{s\in S} E[T_s]\cdot\delta_{w,k,s} \quad (25)$$

$$Var[PT_{w,k}] = \sum_{s\in S} Var[T_s]\cdot\delta_{w,k,s} + 2\cdot\sum_{s1\in S}\sum_{s2(\neq s1)\in S}\delta_{w,k,s1}\cdot\delta_{w,k,s2}\cdot Cov[T_{s1}, T_{s2}] \quad (26)$$

By considering risk-averse driving behavior (Sumalee and Xu [27]), the path travel cost $c_{w,k}$ in this study is defined as:

$$c_{w,k} = E[PT_{w,k}] + \gamma\cdot\sqrt{Var[PT_{w,k}]} \quad \forall w\in W, \forall k\in K_w \quad (27)$$

where γ is a positive parameter.

3 Multiclass traffic assignment model

In this section, we formulate a multiclass traffic assignment model under the stochastic mixed flow of AVs and HVs. This model introduces the stochastic network variables formulated in the previous section. Moreover, we formulate an optimal AV CH-setting problem by utilizing the proposed traffic assignment model.

It is reasonable to suppose that AVs can acquire more precise travel time information than HVs can acquire, since the AVs in this study are connected to each other by ICT technology. In this study, the path choice behavior of HVs is expressed by the logit-based stochastic user equilibrium (SUE) principle, whereas that of AVs is expressed by the user equilibrium (UE) principle. Without a loss of generality, the UE principle applied to AV path choice behavior can be replaced by the SUE principle with less precision of path travel cost information than that for HVs. According to Wang et al. [5], the generalized path travel cost for HV is defined as follows:

$$g_{h,w,k} = c_{w,k} + \frac{1}{\theta} \cdot \ln\left(\frac{f_{h,w,k}}{q_{h,w}}\right) \quad \forall w \in W, \forall k \in K_w \tag{28}$$

where $f_{h,w,k} = E[F_{h,w,k}]$ and $q_{h,w} = E[Q_{h,w}]$. θ in (28) is a dispersion parameter of the SUE assignment, and it shows that the larger the parameter is, the more precise the path travel cost information is. The equilibrium conditions for OD pair w are that the generalized path travel costs for used paths $g_{h,w,k}$ all be all equal. Additionally, $g_{h,w,k}$ of used paths are less than or equal to those of unused paths. These conditions are formulated as the following variational inequality problem (VIP) (Nagurney [30]):

$$\sum_{w \in W} \sum_{k \in K_w} g_{h,w,k} \cdot \left(f_{h,w,k} - f_{h,w,k}^*\right) \geq 0 \tag{29}$$

s.t.

$$\sum_{k \in K_w} f_{h,w,k} = q_{h,w} \ \forall w \in W \tag{30}$$

$$f_{h,w,k} \geq 0 \ \forall w \in W, \forall k \in K_w \tag{31}$$

In (29), $f_{h,w,k}^*$ is the $f_{h,w,k}$ at the equilibrium state. The path choice behavior of AVs that is expressed by the UE principle is formulated as the following VIP:

$$\sum_{w \in W} \sum_{k \in K_w} c_{w,k} \cdot \left(f_{a,w,k} - f_{a,w,k}^*\right) \geq 0 \tag{32}$$

s.t.

$$\sum_{k \in K_w} f_{a,w,k} = q_{a,w} \ \forall w \in W \tag{33}$$

$$f_{a,w,k} \geq 0 \ \forall w \in W, \forall k \in K_w \tag{34}$$

From (29) and (34), the traffic assignment problem for the mixed flow of HVs and AVs can be formulated as the following VIP:

$$\sum_{w \in W} \sum_{k \in K_w} g_{h,w,k} \cdot \left(f_{h,w,k} - f_{h,w,k}^*\right) + \sum_{w \in W} \sum_{k \in K_w} c_{w,k} \cdot \left(f_{a,w,k} - f_{a,w,k}^*\right) \geq 0 \tag{35}$$

s.t. (30), (31), (33), and (34)

Note that at an AV market penetration rate of p_a, the corresponding AV penetration rate for each link ω_s^a in (12a) is endogenously calculated from (35) and the corresponding path-link incident matrices. Therefore, there is no need to set the AV penetration rate exogenously to each link in the network.

4 Numerical experiments

In this section, we show the results of numerical experiments carried out to demonstrate the proposed models in a test network.

We analyze the impact of the AV penetration rate on the change in total travel time of the entire road network. The total travel time is represented as stochastic variable corresponding to link flow and link travel time defined above shown as:

$$TT_s = V_s \cdot T_s = V_s \cdot t_s^0 + V_s \cdot T_s' \quad \forall s \in S \tag{36}$$

The mean and variance of total link travel time are respectively represented as:

$$E[TT_s] = E[V_s \cdot T_s] = t_s^0 \cdot E[V_s] + E[V_s \cdot T_s'] \tag{37}$$

$$Var[TT_s] = Var[V_s \cdot T_s] = (t_s^0)^2 \cdot Var[V_s] + Var[V_s \cdot T_s'] + 2 \cdot t_s^0 \cdot Cov[V_s, V_s \cdot T_s'] \tag{38}$$

The total travel time for all drivers in the network is calculated as:

$$TT = \sum_{s \in S} TT_s \tag{39}$$

Its mean and variance are represented respectively as:

$$E[TT] = E\left[\sum_{s \in S} TT_s\right] = \sum_{s \in S} E[TT_s] \tag{40}$$

$$Var[TT] = \sum_{s \in S} Var[TT_s] + 2 \cdot \sum_{s1 \in S} \sum_{s2 (\neq s1) \in S} Cov[TT_{s1}, TT_{s2}] \tag{41}$$

We use the Sioux Falls network shown in Figure 3. We define three origin nodes and three destination nodes. This network has nine OD pairs, and each OD pair has four paths. The free-flow travel times for links 1, 37, 38, 56, and 60 are set as 15 [min], and those for the others are set as 5 [min]. The parameters of the BPR function for all links, α_s and β_s, are set as 0.15 and 4, respectively. The mean and the coefficient of variation for total traffic demand are respectively set as 18,000 [pcu/hour] and 0.10 ($Q \sim LN(9.79, 9.95 \times 10^{-3})$). Each mean OD demand is set as 2,000 [pcu/hour] (OD demand $Q_w \sim LN(7.60, 9.95 \times 10^{-3})$). In this analysis, we set the parameter γ in (27) as 1.0. We set the dispersion parameter θ in (28) as 0.1. In setting the CH for each vehicle, it is reasonable to suppose that the link travel cost, $E[T_s] + \gamma \cdot \sqrt{Var[T_s]}$, increases as the link flow increases, regardless of the vehicle type. However, this relationship does not always hold depending on the parameters of the distributions. Appendix C shows the conditions of the parameters that result in the monotonicity of the link cost function in the stochastic mixed flow. Based on the results of Appendix C, we set h_0, μ_{R_h}, and $\sigma_{R_h}^2$ as 2.0 [sec], 0.14 [sec], and 0.04 [sec^2], respectively. The corresponding mean HV CH is calculated as 2.35 [sec]. We set $\sigma_{R_a}^2$ as 0.03 [sec^2]. Figure 4 shows the total travel cost in the network, $E[TT] + \gamma \cdot \sqrt{Var[TT]}$, when the market penetration rate of AVs, p_a in (2), and $E[R_a]$ vary. Figure 5 shows the probability density functions (pdf) of the path flows between OD pair 1, which is from node 1 to 7, when the mean AV CH is 2.01 [sec] and $p_a = 0.5$.

From Figure 4, it is known that the total travel cost decreases with a decrease in the mean CH or with an increase in the AV penetration rate in the network. The general trend is that the mean travel times of the paths that carry AV flow are shorter than those that carry no AV flow. This is caused by the mean link capacity increase brought by the AV flow. The spread of AVs ultimately contributes to reductions in the total travel cost in the network. Figure 5 shows the path flow distributions of O-D pair 1, which is from node 1 to node 7 and has four paths. The link path sequence of paths 1-4 of O-D pair 1 are respectively represented as 1-4-16-20, 2-6-9-12-16, 2-6-9-13-24-20, and 1-4-15-13-24-20. As shown in Figure 5, it is known that the variance of path flow increases as the mean path flow increases. This relationship shown in the figure is consistent with (10). The difference in path choice patterns of AVs and HVs can result from the differences in the exogenous assumptions (i.e., path choice criteria) and the parameters (i.e., the AV penetration ratios of O-D pairs; the parameters of headway of AVs and link capacity model). For example, in general, vehicles following SUE will also choose paths with non-minimal travel times that vehicles following UE will not choose due to the assumption of perception errors. Figure 6 compares the path flows of $\gamma = 0$ and $\gamma = 1$. Differences in drivers' risk preferences affect their recognition of path choice costs and path choice behavior.

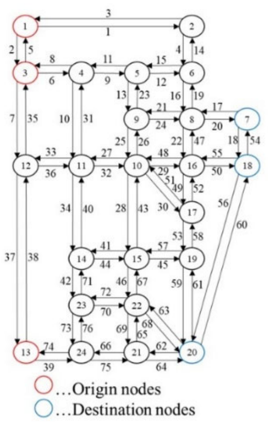

Figure 3. Test network

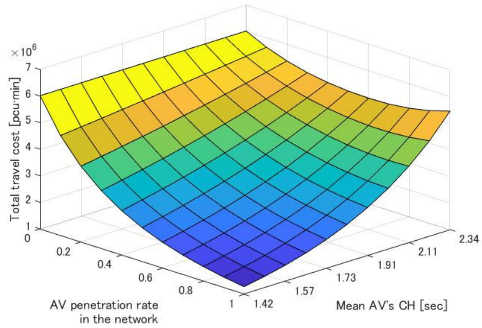

Figure 4. Total travel cost

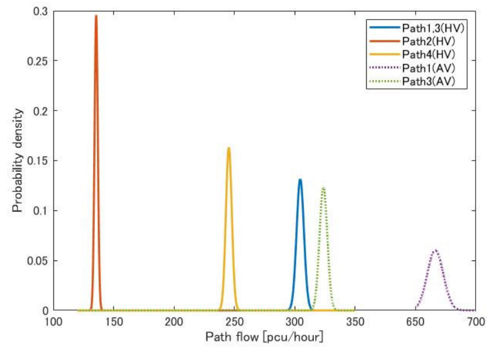

Figure 5. Path flow distributions of O-D pair 1

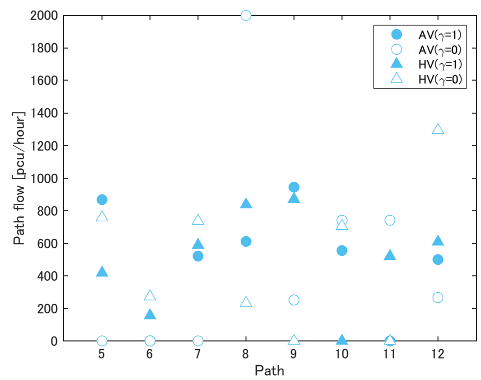

Figure 6. Mean path flows of risk-neutral case and risk-averse case

5 Concluding remarks

This study proposed a multiclass traffic assignment model under the stochastic mixed traffic flows of AVs and HVs. A stochastic link capacity model was introduced to these models. The mean and variance of the stochastic link capacity proposed in this study is expressed as a function of the AV penetration rate for the link. The proposed model introduces stochastic variables to represent travel time uncertainty. We assume that the mean and standard deviation of travel time affect route choices in the road network. To verify the proposed multiclass traffic assignment model, we performed numerical calculations in a test network. The results showed that when the AV penetration increases, the total travel cost of the whole road network decreases. Additionally, it is shown that the improvement of AV CH brings about a reduction in total travel cost.

For the future works, firstly, we will analyze the properties of other path choice principles. As shown in the literature, various path choice principles have been proposed for each vehicle type. We need to analyze transitions of travel cost when we assume different principles. Secondly, the effect of the stochastic platoon order can be considered in link capacity model. One of the authors proposed the way to consider stochastic platoon order in static equilibrium framework (Hu and Kato, [31]). Thirdly, the proposed network equilibrium model can be used for the policy analysis in the AV era. For example, the parking choice behavior of AVs in congested areas is an interesting problem to be formulated as a network equilibrium problem. A limitation of the model proposed in this study is that it does not take these into account in setting the AV CH. Fourthly, the relationship between the equilibrium solutions and the initial solutions should be studied more. An equilibrium solution of the traffic assignment problem considering the mixed flow depends on the initial solution and the parameters of link capacity model and link cost function. When applying the traffic assignment model to a real road network, the variability of the equilibrium solutions depending on the initial solutions should be investigated like a previous study (e.g., [6]). For the applicability of the proposed model to the real road network, the impact of road network and AV performances on the characteristics of equilibrium solutions should be clarified. Finally, the causes of the differences in route choice patterns of AVs and HVs need to be studied using a quantitative rather than qualitative approach. Identifying the causes of the differences is meaningful to develop the method to guide the routes of AVs or HVs by setting the congestion charge or the road space dedicated for AVs.

List of Abbreviations AV: Autonomous Vehicle; HV: Human-driven Vehicle; CH: Critical Headway; UE: User Equilibrium; SUE: Stochastic User Equilibrium.

Declarations

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Appendix A: Notations

W	Set of origin–destination (OD) pairs
K_w	Set of paths serving OD pair w
S	Set of links
Q	Stochastic total traffic demand
μ_X, σ_X	Parameters of random variable X following a lognormal distribution
Q_z	Stochastic total traffic demand for type $z \in \{h, a\}$ vehicles where h and a represent HV and AV, respectively
p_z	Penetration rate of type z vehicles
$Q_{z,w}$	Stochastic traffic demand of type z vehicles between OD pair w
$q_{z,w}$	Mean of stochastic traffic demand of type z vehicles between OD pair w
$p_{z,w}$	Ratio of traffic demand of type z vehicles to total traffic demand of the same type of vehicles between OD pair w
$F_{z,w,k}$	Stochastic flow of type z vehicles on path k between OD pair w
$f_{z,w,k}$	Mean of stochastic flow of type z vehicles on path k between OD pair w
$p_{z,w,k}$	Ratio of the flow of type z vehicles on path k between OD pair w to total traffic demand of the same type vehicles and OD pair
$V_{z,s}$	Stochastic flow of type z vehicles on link s
$v_{z,s}$	Mean of stochastic flow of type z vehicles on link s
$\delta_{w,k,s}$	Variable that equals 1 if link s is part of path k between OD pair w and 0 otherwise
$\hat{p}_{z,s}$	Ratio of the flow of type z vehicles on link s to total traffic demand
V_s	The total link flow for both vehicle types
$\hat{p}_s$	Ratio of the flow on link s to total traffic demand
h_0	Basic CH for all vehicles
R_z	Stochastic ratio of the CH for type z vehicles to the basic CH
$\omega_{a,s}$	AV penetration rate on link s
r_z^i	The ith sampled CH ratio for vehicle type z
r_s	The geometric mean of the sequence of the sampled CH ratios on link s
$\hat{r}_{z,s}$	The product of all samples of r_z^i on link s
$n_{z,s}$	The sample number for vehicle z on link s
n_s	The sample number on link s
R_s	Stochastic ratio of the CH for the mixed flow of AVs and HVs to the basic CH on link s
$\hat{R}_{z,s}$	The $n_{z,s}$ order moment of R_s

H_s	Stochastic CH for vehicles on link s
C_s	Stochastic capacity of link s
T_s	Stochastic travel time of link s
t_s^0	Free-flow travel time of link s
α_s, β_s	Parameters of BPR function of link s
T_s'	Stochastic delay time due to congestion on link s
$PT_{w,k}$	Stochastic travel time of path k between OD pair w
$c_{w,k}$	Travel cost of path k between OD pair w
γ	Parameter of risk-aversion degree
TT_s	Total travel time of link s
TT	Total travel time of the entire road network
$g_{h,w,k}$	Generalized travel cost of path k between OD pair w defined for HV
θ	Dispersion parameter of logit model

Appendix B: Mathematical properties of link capacity

From (16), the mean of link capacity $E[C_s]$ is represented as:

$$E[C_s] = \frac{3600}{h_0} \cdot exp\begin{pmatrix} -\omega_{a,s} \cdot \mu_{R_a} - \left(1-\omega_{a,s}\right) \cdot \mu_{R_h} \\ +\frac{1}{2}\left\{\left(\omega_{a,s}\right)^2 \cdot \sigma_{R_a}^2 + \left(1-\omega_{a,s}\right)^2 \cdot \sigma_{R_h}^2\right\} \end{pmatrix} \tag{42}$$

From $\frac{\partial E[C_s]}{\partial \omega_{a,s}} = 0$, the AV penetration rate minimizing $E[C_s]$ is given by:

$$\omega_{a,s}^* = \frac{\mu_{R_a} - \mu_{R_h} + \sigma_{R_h}^2}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{43}$$

The resultant distribution parameters of C_s are then:

$$\mu_{C_s}^* = ln(3600) - ln\,(h_0) - \frac{\begin{pmatrix} \mu_{R_a} \cdot \left(\mu_{R_a} - \mu_{R_h} + \sigma_{R_h}^2\right) \\ +\mu_{R_h} \cdot \left(-\mu_{R_a} + \mu_{R_h} + \sigma_{R_a}^2\right) \end{pmatrix}}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{44}$$

$$\sigma_{C_s}^{*\,2} = \sigma_{R_s}^{*\,2} = \frac{\sigma_{R_a}^2 \cdot \sigma_{R_h}^2 + \left(\mu_{R_a} - \mu_{R_h}\right)^2}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{45}$$

The coefficient of variation of a stochastic variable is defined as the ratio of the standard deviation and the mean. $CV[C_s]$ is then defined as:

$$CV[C_s] = \frac{SD[C_s]}{E[C_s]} \tag{46}$$

where

$$SD[C_s] = \sqrt{Var[C_s]} = E[C_s] \cdot \sqrt{\exp\left(\sigma_{C_s}^2\right) - 1} \tag{47}$$

Substituting (47) into (46), we obtain:

$$CV[C_s] = \sqrt{\exp\left(\left(\omega_{a,s}\right)^2 \cdot \sigma_{R_a}^2 + \left(1-\omega_{a,s}\right)^2 \cdot \sigma_{R_h}^2\right) - 1} \tag{48}$$

From $\frac{\partial CV[C_s]}{\partial \omega_s^a} = 0$, the AV penetration rate at which $CV[C_s]$ is minimized is given by:

$$\omega_{a,s}^+ = \frac{\sigma_{R_h}^2}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{49}$$

The resultant distribution parameters of C_s are then:

$$\mu_{C_s}^+ = \ln(3600) - \ln\,(h_0) - \frac{\mu_{R_a} \cdot \sigma_{R_h}^2 + \mu_{R_h} \cdot \sigma_{R_a}^2}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{50}$$

$$\sigma_{C_s}^{+\,2} = \sigma_{R_s}^{+\,2} = \frac{\sigma_{R_a}^2 \cdot \sigma_{R_h}^2}{\sigma_{R_a}^2 + \sigma_{R_h}^2} \tag{51}$$

In (49), if R_a is a stochastic variable, namely $\sigma_{R_a}^2 > 0$, then there exists an AV penetration rate $\omega_{a,s}^+ (> 0)$ that minimizes $CV[C_s]$. If R_a is a deterministic variable, namely $\sigma_{R_a}^2 = 0$, then $CV[C_s]$ monotonically decreases in $\omega_{a,s} \in [0, 1]$.

It is impossible to derive an analytical expression of $\omega_{a,s}$ that minimizes the standard deviation of link capacity $\sqrt{Var[C_s]}$. Therefore, we will discuss next the existence range of $\omega_{a,s}$ which minimizes the standard deviation of link capacity. We consider first a case when $\mu_{R_h} - \mu_{R_a} > 0$, i.e., $\omega_{a,s}^* < \omega_{a,s}^+$. Hereinafter, $\sqrt{Var[C_s]}$, $E[C_s]$, and $\frac{\sqrt{Var[C_a]}}{E[C_a]}$ are represented as $f(\omega_{a,s})$, $g(\omega_{a,s})$, and $h(\omega_{a,s})$, respectively. $f(\omega_{a,s})$, $g(\omega_{a,s})$, and $h(\omega_{a,s})$ hold the following relationship:

$$f\left(\omega_{a,s}\right) = g\left(\omega_{a,s}\right) \cdot h\left(\omega_{a,s}\right) \tag{52}$$

The first derivative of $f(\omega_{a,s})$ with respect to $\omega_{a,s}$ is calculated as:

$$f'^{(\omega_{a,s})} = g'\left(\omega_{a,s}\right) \cdot h\left(\omega_{a,s}\right) + g\left(\omega_{a,s}\right) \cdot h'\left(\omega_{a,s}\right) \tag{53}$$

where $f' = \frac{df}{d\omega_{a,s}}$, $g' = \frac{dg}{d\omega_{a,s}}$, and $h' = \frac{dh}{d\omega_{a,s}}$. Note that $g(\omega_{a,s})$ and $h(\omega_{a,s})$ are non-negative convex functions and hold $g'(\omega_{a,s}^*) = 0$ and $f'(\omega_{a,s}^+) = 0$. Therefore, $g'(\omega_{a,s}) \cdot h(\omega_{a,s})$ is a continuous increasing function that holds $g'(\omega_{a,s}) \cdot h(\omega_{a,s}) = 0$. Also, $g(\omega_{a,s}) \cdot h'(\omega_{a,s})$ is a continuous increasing function that holds $g(\omega_{a,s}^+) \cdot h'(\omega_{a,s}^+) = 0$. From the above discussion, it is shown that $f'(\omega_{a,s})$ is a continuous increasing function. Since $f'(\omega_{a,s}^*) < 0 < f'(\omega_{a,s}^+)$, there exists $\omega_{a,s}$ such that $f'(\omega_{a,s}) = 0$ and $\omega_{a,s} \in (\omega_{a,s}^*, \omega_{a,s}^+)$. Namely, there exists $\omega_{a,s} \in (\rho_{a,s}^*, \omega_{a,s}^+)$ that minimizes the standard deviation of link capacity. In the same manner, when $\mu_{R_h} - \mu_{R_a} < 0$, i.e., $\omega_{a,s}^* > \omega_{a,s}^+$, the results of $f'(\omega_{a,s}) = 0$ and $\omega_{a,s} \in (\omega_{a,s}^+, \omega_{a,s}^*)$ are obtained.

Appendix C: Monotonicity of link travel cost

The increase in AV penetration rate increases the traffic capacity of a link. However, it is reasonable to suppose that the link travel cost increases monotonically with respect to link flow in a road network. We will derive parameter-setting conditions for AV CH that guarantee that the link travel cost increases monotonically with respect to the link flow. We do not address the case of $\mu_{R_h} - \mu_{R_a} < 0$. When $\mu_{R_h} - \mu_{R_a} < 0$, the mean AV CH can be larger than the mean HV CH even if the standard deviation of AV CH is less than that of HV CH. However, such parameter setting of AVs is irrational since it does not motivate road users to use AVs. Therefore, we only address the case of $\mu_{R_h} - \mu_{R_a} \geq 0$.

We will find conditions that ensure that the mean link travel time increases monotonically with respect to the mean link flow of AV, $v_{a,s} = E[V_{a,s}]$. Hereinafter, the mean values of $V_{z,s}$, V_s, and C_s are represented as $v_{z,s}$, v_s, and c_s, respectively. In the following discussion, the variables are μ_{R_a} and $\sigma_{R_a}^2$, i.e., the other parameters such as β_s, μ_{R_h}, and $\sigma_{R_h}^2$ are now assumed as given calibration parameters. The partial derivative of mean link travel time with respect to $v_{a,s}$ is represented as:

$$\frac{\partial E[T_s]}{\partial v_{a,s}} = E\left[\frac{\partial T_s}{\partial V_s} \cdot \frac{\partial V_s}{\partial V_{a,s}}\bigg|_{V_{a,s}=v_{a,s}}\right] + E\left[\frac{\partial T_s}{\partial C_s} \cdot \frac{\partial C_s}{\partial V_{a,s}}\bigg|_{V_{a,s}=v_{a,s}}\right] \tag{54}$$

Although some approximate computation of random variables is required in the process of computing the right-hand side of (54), the condition under which the mean of link travel time increases monotonically with respect to the link flow can be obtained under the approximate computation as follows:

$$1 > (\mu_{R_h} - \mu_{R_a}) \tag{55}$$

Next, we will find conditions that ensure that the variance of the link travel time increases monotonically with respect to link AV flow $v_{a,s}$. The partial derivative of mean link travel time with respect to $v_{a,s}$ is represented as:

$$\frac{\partial Var[T_s]}{\partial v_{a,s}} = \frac{\partial Var[T_s']}{\partial V_{a,s}}\bigg|_{V_{a,s}=v_{a,s}} = \frac{\partial E[(T_s')^2]}{\partial V_{a,s}}\bigg|_{V_{a,s}=v_{a,s}} - 2E[T_s'] \cdot \frac{\partial E[T_s']}{\partial v_{a,s}} \tag{56}$$

Suppose (55) and $\beta_s \geq 1$ holds, then the conditions that ensure that the variance of link travel time $Var[T_s]$ increases monotonically with respect to $v_{a,s}$ are represented as:

$${\sigma_{R_s}^{+}}^2 > \left(\frac{2}{\beta_s} - 1\right) \cdot \sigma_Q^2 \tag{57}$$

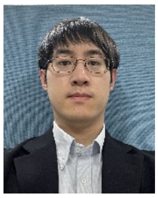
Finally, from (55) and (57), the monotonicity condition of link cost can be represented as:

$$1 > (\mu_{R_h} - \mu_{R_a}) \; and \; {\sigma_{R_s}^{+}}^2 > \left(\frac{2}{\beta_s} - 1\right) \cdot \sigma_Q^2 \tag{58}$$

References

1. Fagnant, D.J., Kockelman, K.: Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations. Trans. Res. A, 77, 167-181, (2015). https://doi.org/10.1016/j.tra.2015.04.003
2. Levin, M. W., Boyles, S. D.: A multiclass cell transmission model for shared human and autonomous vehicle roads. Transp. Res. C, 62, 103–116, (2016). https://doi.org/10.1016/j.trc.2015.10.005
3. Seo, T., Asakura, Y.: Endogenous market penetration dynamics of automated and connected vehicles: Transport-oriented model and its paradox. Transp. Res. Proc., 27, 238-245, (2017). https://doi.org/10.1016/J.TRPRO.2017.12.028
4. Liu, Z., Song, Z.: Strategic planning of dedicated autonomous vehicle lanes and autonomous vehicle/toll lanes in transportation networks, Transp. Res. C, 106, 381-403, (2019). https://doi.org/10.1016/J.TRC.2019.07.022
5. Wang, J., Peeta, S., He, X.: Multiclass traffic assignment model for mixed traffic flow of human-driven vehicles and connected and autonomous vehicles. Trans. Res. B, 126, 139–168, (2019). https://doi.org/10.1016/j.trb.2019.05.022
6. Wang, J., Lu, L., Peeta, S., He, Z.: Optimal toll design problems under mixed traffic flow of human-driven vehicles and connected and autonomous vehicles. Transp. Res. C, 125, (2021). https://doi.org/10.1016/j.trc.2020.102952
7. Wang, J., Wang, W., Ren, G., Yang, M.: Worst-case traffic assignment model for mixed traffic flow of human-driven vehicles and connected and autonomous vehicles by factoring in the uncertain link capacity. Transp. Res. C, 140, (2022). https://doi.org/10.1016/J.TRC.2022.103703
8. Tani, R., Sumalee, A., Uchida, K.: Travel time reliability-based optimization problem for CAVs dedicated lanes. Transportmetrica A: Trans. Sci., (2021). https://doi.org/10.1080/23249935.2021.1954107
9. Levin, M.W., Boyles, S.D.: Effects of autonomous vehicle ownership on trip, mode, and route choice. Transp. Res. Rec., 2493, 29–38, (2015). https://doi.org/10.3141/2493-04
10. Ghiasi, A., Hussain, O., Qian, Z. (Sean), Li, X.: A mixed traffic capacity analysis and lane management model for connected automated vehicles: A Markov chain method. Trans. Res. B, 106, 266–292, (2017). https://doi.org/10.1016/j.trb.2017.09.022
11. Zhou, J., Zhu, F.: 2020. Modeling the fundamental diagram of mixed human-driven and connected automated vehicles. Transp. Res. C, 115, (2020). https://doi.org/10.1016/J.TRC.2020.102614
12. Guo, Q., Ban, X. (Jeff), Aziz, H. M. A.: Mixed traffic flow of human driven vehicles and automated vehicles on dynamic transportation networks. Transp. Res. C, 128, (2021). https://doi.org/10.1016/j.trc.2021.103159
13. Chen, S., Wang, H., Xiao, L., Meng, Q.: Random capacity for a single lane with mixed autonomous and human-driven vehicles: Bounds, mean gaps and probability distributions. Trans. Res. E 160, (2022). https://doi.org/10.1016/J.TRE.2022.102650
14. Greenshields, B.D.: A study of traffic capacity. Highway Research Board Proceedings, 14, (1935).
15. van den Berg, V. A. C., Verhoef, E. T.: Autonomous cars and dynamic bottleneck congestion: The effects on capacity, value of time and preference heterogeneity. Trans. Res. B, 94, 43–60, (2016). https://doi.org/10.1016/j.trb.2016.08.018
16. Le Vine, S., Kong, Y., Liu, X., Polak, J.: Vehicle automation and freeway 'pipeline' capacity in the context of legal standards of care. Transp., 46 (4), (2019). https://doi.org/10.1007/s11116-017-9825-8
17. Chen, A., Yang, H., Lo, H. K., Tang, W. H.: Capacity reliability of a road network: An assessment methodology

and numerical results. Trans. Res. B, 36(3), 225–252, (2002). https://doi.org/10.1016/S0191-2615(00)00048-5
18. Uchida, K.: Travel time reliability estimation model using observed link flows in a road network. Comput.-Aided Civ. Infrastruct. Eng., 30 (6), 449–463, (2015). https://doi.org/10.1111/mice.12109
19. Bagloee, S. A., Sarvi, M., Patriksson, M., Rajabifard, A.: A mixed user-equilibrium and system-optimal traffic flow for connected vehicles stated as a complementarity problem. Comput.-Aided Civ. Infrastruct. Eng., 32 (7), 562–580, (2017). https://doi.org/10.1111/mice.12261
20. Zhang, K., Nie, Y. (Marco).: Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. Transp. Res. C, 87, 75–90, (2018). https://doi.org/10.1016/j.trc.2017.12.011
21. Chen, Z., He, F., Zhang, L., Yin, Y.: Optimal deployment of autonomous vehicle lanes with endogenous market penetration. Transp. Res. C, 72, 143-156, (2016). https://doi.org/10.1016/J.TRC.2016.09.013
22. Nakayama, S., Takayama, J.: Traffic network equilibrium model for uncertain demands. Proceedings of the 82nd Transportation Research Board Annual Meeting, April, 1–9, (2003).
23. Clark, S., Watling, D.: Modelling network travel time reliability under stochastic demand. Trans. Res. B, 39(2), 119–140, (2005). https://doi.org/10.1016/j.trb.2003.10.006
24. Lam, W. H. K., Shao, H., Sumalee, A.: Modeling impacts of adverse weather conditions on a road network with uncertainties in demand and supply. Trans. Res. B, 42(10), 890–910, (2008). https://doi.org/10.1016/j.trb.2008.02.004
25. Uchida, K.: Estimating the value of travel time and of travel time reliability in road networks. Trans. Res. B, 66, 129–147, (2014). https://doi.org/10.1016/j.trb.2014.01.002
26. Zhou, Z. and Chen, A.: Comparative analysis of three user equilibrium models under stochastic demand. J. Adv. Transp., 42(3), 239-263, (2008). https://doi.org/10.1002/atr.5670420304
27. Sumalee, A., Xu, W.: First-best marginal cost toll for a traffic network with stochastic demand. Trans. Res. B, 45(1), 41–59, (2011). https://doi.org/10.1016/j.trb.2010.04.007
28. Uno, N., Kurauchi, F., Tamura, H., Iida Y.: Using bus probe data for analysis of travel time variability, J. Intell. Transp. Syst., 13 (1), 2-15, (2009). https://doi.org/10.1080/15472450802644439
29. Srinivasan, K. K., Prakash, A. A., Seshadri, R.: Finding most reliable paths on networks with correlated and shifted log-normal travel times. Trans. Res. B, 66, 110–128, (2014). https://doi.org/10.1016/j.trb.2013.10.011
30. Nagurney, A.: A multiclass, multicriteria traffic network equilibrium model. Math. Comput. Model., 32(3–4), 393–411, (2000). https://doi.org/10.1016/S0895-7177(00)00142-4
31. Hu, Z., Kato, T.: Traffic assignment model for mixed flow of UE-principled human driven vehicles and SO-principled autonomous vehicles with stochastic link capacity. The 26th International Conference of Hong Kong Society for Transportation Studies, 2023.



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