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Experimental verification of model-free active damping system based on virtual controlled object and fuzzy sliding mode control

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Abstract

To actively suppress vibrations in mechanical structures, this research presents a novel fuzzy sliding mode controller that can be designed without involving mathematical plant modeling and any plant's parameters. The proposed strategy introduces a virtual controlled object (VCO)-based model-free scheme into a sliding mode control (SMC) system while addressing chattering mitigation by adaptive fuzzy inference. A framework of the model-free design is first formulated by interposing the VCO between a real controlled structure and an actuator. Based on a simple controlled frequency band-based design policy, the VCO, which is represented by a single-degree-of-freedom (SDOF) system, is specified to enable model-free active controller design. The VCO-based design process is then combined with SMC, and a Lyapunov function approach is employed to analyze the convergence. Furthermore, the chattering issue, which hinders successful implementation of SMC, is explicitly addressed by employing a fuzzy inference system. The fuzzy rules adjust the control effort online to ensure rapid convergence to the sliding surface while avoiding drastic switching of the control input, in accordance with variations of the switching function. Finally, experimental verifications are performed by applying the proposed vibration controller to a cantilever plate with characteristic perturbations. The verification results demonstrate that the proposed fuzzy SMC provides sufficient vibration reduction and realizes high robustness to changes in the controlled object.

Keywords: Model-free control, Active damping, Virtual controlled object, Sliding mode control, Proof-mass actuator, Fuzzy inference

1. Introduction

Vibrations deteriorate the precision and comfort of mechanical systems and induce fatigue failures, shortening the service life of structures. Therefore, vibration control technologies are indispensable to improve the performance of mechanical systems [1][2][3], and many efforts have been dedicated to developing effective approaches [4][5][6]. Their significance has been increased by the recent trend toward lighter, smaller machines and growing popularity of flexible structures. Active vibration control (AVC) has attracted a great deal of attention because of its capability to realize stronger damping effects for complicated mechanical systems [7][8][9]. In particular, AVCs using inertial-mass actuators or active mass dampers, which have SDOF structure-based working principles, have been widely employed. The application examples of such AVC include not only mechanical systems such as industrial robots used in precision machining process [10], but also civil structures such as lightweight footbridges [11]. Hence, development of their application-oriented advanced control strategy has been important. It should be noted that controlled objects have complicated mechanisms and dynamical properties with uncertainty in real-world industrial applications; therefore, AVC, which adaptively suppresses vibrations by utilizing the mechanical energy supplied from the outside, is more suitable for providing powerful damping effects for such intricate mechanical and infrastructure systems.

In general, however, AVC involves mathematical plant modeling, such as finite element method and experimental modal analysis. In other words, AVC belongs to model-based control (MBC) in which the modeling and identification of the controlled object's dynamics or specifications are needed, increasing development time and costs. Accurate modeling of complicated mechanical and infrastructure (building structure) systems in real-world industrial applications is challenging, and such modeling task imposes a large burden and difficulty on designers. Moreover, MBC is easily affected by characteristic variations always present in real controlled structures because of reliance on a specific configuration in the modeling process. Therefore, a model-based controller designed for a certain structure can be no longer applied to another different structure. AVC design that does not require modeling of a controlled structure and parameter identification is useful and effective to pioneer the application area of AVC in industrial worlds. Consequently, the development of such AVCs, i.e., model-free controls (MFCs) is needed.

MFCs have been established in numerous prior studies to eliminate need for mathematical modeling of controlled objects. To this end, using artificial intelligence tools like neural networks (NNs) is one of the most effective tactics [12][13]. To derive the direct inverse controller, a representative approach involves identification of an inverse plant model via an artificial neural network [14]. One study formulated a training procedure for a NN-based active controller for flexible cantilever plates [15]. Radial basis function NNs have been widely employed in advanced active oscillation controls [16][17][18]. To learn suitable control actions, however, most NN-based MFCs require us to collect a huge amount of training data as well as waiting for completing the training process. This requirement may present a high hurdle to convenient implementation.

Another powerful approach which does not need exact plant models is active disturbance rejection

control (ADRC) [19]. In ADRC, all plant uncertainties such as external disturbances and unknown dynamics are lumped into a total disturbance [19][20]. The total disturbance is then canceled out by estimation through an extended state observer. Many works [21] have benefited from ADRC, covering active vibration reduction [22]. One unique application area of ADRC extends to the active control of combustion oscillation [23]. However, depending on the plant structure, the formulation of the lumped disturbance, which requires deep insight into the plant characteristics, is not necessarily simple, making the design process of ADRC complicated. In addition to the need to obtain the relative degree of a plant, which is not straightforward, the robustness largely depends on setting an appropriate observer bandwidth, which is a difficult task.

Thus, a straightforward and useful model-free active vibration controller has yet to be created. Inspired by the aforementioned context, a novel model-free design scheme of an active vibration controller was presented recently, which utilizes a virtual controlled object (VCO) [24], referred to as VCO-based model-free control (VCO-MFC). The concept is simple but reasonable; instead of using a real controlled object, a vibration controller is designed based on only the VCO and an actuator. That is, a model-free design is allowed by simply setting the proper VCO. The high robustness to fluctuations in various controlled objects has been confirmed in many literatures [25][26][27]. One significant advantage of this method is that, after the VCO is properly set, the controller design process of familiar model-based control theories can be followed directly. Therefore, the inheritance of various model-based controllers can be directly delivered to VCO-MFC.

The application area of the VCO-based approach also covers a nonlinear control theory [25][28]. As a representative nonlinear controller, sliding mode control (SMC) is the most familiar approach. This involves switching of the control input depending on system state variations. Its excellent robustness is a remarkable advantage of SMC, which enables applications for the plant's uncertainties and disturbances. One notable work reported that the robustness to plant uncertainties can be further reinforced with a combination of SMC and VCO-MFC [28]. The extendibility of VCO-MFC allows for such unique fusion. In other previous literatures, the effectiveness of SMC has been confirmed for uncertain systems [29][30]. An improved SMC has been also developed such as combination with NNs to address nonlinearities and unknown system dynamics [31][32]. One notable study applied two-hidden-layer recurrent neural networks with superior fitting ability to construct a more robust fractional-order SMC [33]. The successful compensation property for unknown nonlinearities was experimentally proved in controlling an active power filter. From the viewpoint of industrial application of SMCs, its effectiveness has been shown in active suspension systems [34][35]. Such applications have promoted the development of various unique techniques including introduction of a disturbance observer [36][37] and time-delay estimation-based control [38]. There are many examples of using SMCs for vibration control [39] such as smart structures for a piezoelectric laminate cantilever beam [40], drill string system operations [41], and soft robots [42]. Nevertheless, detailed models or specifications of the real controlled objects are still necessary in most of the existing SMC design procedures. Although a few previous works constructed model-free approaches [43][44], the shortcomings that hinder practical and simple

implementations remain: a complicated design process with many adjustable parameters, need for an estimator for the unknown plant dynamics, and insufficient management of the chattering problem. In particular, a chattering phenomenon, which always hinders practical applications of SMCs, has serious negative impacts on the controller implementation such as damages to hardware systems, deteriorations of a closed-loop stability, residual high-frequency oscillations, and wasteful consumption of control powers. Therefore, it is needed to effectively compensate for the chattering issues, leading to the enhancement of the robustness.

A fuzzy SMC (FSMC) is one of the promising choices to compensate for the shortcomings of vanilla SMCs. The combination of SMC and fuzzy logic is a reasonable idea because of the capability of fuzzy logic to give flexibility and robustness to control system design based on human-like knowledge. Many types of FSMC have been proposed for complex mechanical systems [45–48], including vibration control problems [49]. Using a bioinspired reference model, an adaptive FSMC has been constructed for vehicle active suspension systems [50] considering the input effects of dead zone and saturation. However, it is required to solve complicated equations of the bioinspired nonlinear dynamics, increasing the design difficulty. Other examples of FSMCs for suspension systems can be found in the literature [51][52]. The introduction of a fuzzy neural network strategy into design of SMC is a powerful way that can approximate and compensate for unknown system uncertainties (i.e., unmodeled dynamics, nonlinearities, and external disturbances), as demonstrated by the improved versions such as the utilizations of the double-loop fuzzy neural network [53] and interval type-2 fuzzy neural network [54]. Thanks to the benefits, it is unnecessary to rely too much on applied plant models. The application example of [53] is a mass–spring–damper structure of micro gyroscope to which the model-free approach (e.g., VCO-MFC) may also be applicable. Furthermore, to improve FSMC for addressing system nonlinearities and uncertainties, the Takagi-Sugeno (T-S) fuzzy method is also effective [55]. One study applied a variable fractional-order SMC with an adaptive approximation mechanism to control earthquake-caused vibrations in building structures [56]. To construct active vibration control for a two-connected flexible piezoelectric plate, a fuzzy fast terminal SMC has been employed [57]. Especially, one notable usage of fuzzy logic is active reduction of the SMC's chattering phenomena [58]. In most of the existing FSMCs, however, the design process depends on the real controlled objects' models or specifications (e.g., parameters and information on the uncertainty). Thus, the associated loads and design costs may be increased. Moreover, the controller's configuration can be intricate because of the needs of adaptive approximation/estimation schemes [59][60].

In summary, the existing previous vibration control approaches have the following difficulties (shortcomings).

- AVC generally requires a modeling process and parameter identification of an applied controlled object in advance, enlarging the development cost, time and burden of designers.
- Because existing MFCs rely on a learning process based on an enormous amount of training data and approximation mechanisms for unknown dynamical models, their controller design is complicated. Additionally, SMCs and FSMCs have the strong robustness against unknown system

uncertainty; however, most of them need some models or parameters of a controlled structure.

- Even though the VCO-based model-free SMC has been presented in the previous literature [28], the chattering phenomenon is hardly addressed. Such insufficient chattering compensation causes a deterioration of robustness in active damping.

This work presents a model-free active damping controller that simultaneously addresses all of these difficulties. To tackle the above existing shortcomings, a novel fuzzy SMC is developed in this study, namely VCO-based model-free fuzzy SMC (VCO-MFFSMC), which allows for the straightforward design of a model-free active vibration controller. The proposed approach is built upon the new combination of the VCO-MFC, SMC, and a fuzzy inference-based compensation for chattering mitigation. Although there have been many existing approaches on fuzzy SMCs as reviewed above, this work is the first attempt to formulate a fuzzy SMC in the framework of VCO-MFC. While the proposed approach eliminates the needs for costly modeling of a controlled structure and parameter identification by the application of VCO-MFC, the chattering mitigation, which has not been adequately addressed in the previous work [28], is simultaneously achieved by the introduction of fuzzy inference. This advancement contributes to reducing a burden in the controller design and improving reliability and safety. The proposed approach is validated thorough an experimental implementation for structural dynamic systems employing a cantilever plate with characteristic variations. The following summarizes the contributions as well as technical novelties of this work.

- (1) The principal contribution of this study is to newly combine all of the three techniques (i.e., VCO-MFC, SMC, and Tsukamoto-type fuzzy inference), therefore creating a novel model-free fuzzy sliding mode vibration controller for active damping of mechanical structures.
- (2) In this study, based on the utilization of VCO-MFC, a model-free fuzzy SMC that does not need modeling of an actual controlled object is more simply realized without relying on complicated approximation/estimation mechanisms for unknown system dynamics, compared to most of existing similar approaches.
- (3) Because model-free control can be realized by simply configuring the VCO, which is a basic SDOF system designed using a controlled frequency band-based inequality, the proposed controller is easily implemented. Thanks to the application of the VCO-MFC concept, we can avoid establishing a time-consuming learning process based on an enormous amount of training data and designing complex approximation/estimation mechanisms for unknown system dynamics, which have been essential in traditional approaches.
- (4) In addition, the robustness of the model-free SMC is considerably improved by successfully compensating for a chattering issue, which cannot be addressed by the existing VCO-SMC of our previous study [28]. The severe chattering is more effectively addressed by introducing adaptive fuzzy-inference-based compensation based on nine linguistic rules. This study mitigates a chattering issue in fuzzy SMC by applying the Tsukamoto-type fuzzy inference to design of active vibration control. To the best of our knowledge, this application example is new and has never been attempted before. The benefit of utilizing the Tsukamoto-type inference is to achieve a reduced computational

load and faster processing in real-time than existing FSMCs.

- (5) Another contribution is to experimentally verify the superior damping performance and chattering mitigation effects of VCO-MFFSMC via comparative experiments using an actual mechanical system (e.g., unknown cantilever plate). The proposed new combination significantly improves the robustness of active damping control.

This paper is organized as follows. In Section 2, model-free controller design based on VCO is described in detail. Section 3 presents a VCO-based sliding mode vibration controller. Further, convergence of the sliding mode control is analyzed using a Lyapunov function technique. In Section 4, fuzzy adaptive compensation based on the Tsukamoto-type inference method is introduced to reduce the chattering phenomenon, resulting in VCO-MFFSMC. In Section 5, the proposed approach is verified through experimental tests for cantilever plates with characteristic variations. Finally, the conclusions are summarized in Section 6.

2. VCO-based model-free vibration control technique (VCO-MFC)

An electromagnetic proof-mass actuator is employed in this work, as indicated in Fig. 1. This model can be described as a SDOF system [24] due to the dynamical principle.

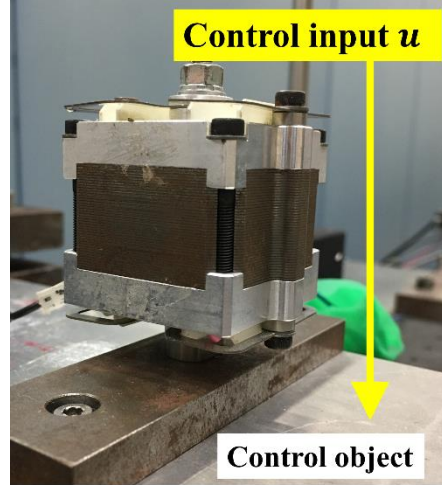


Fig. 1 Proof-mass actuator.

In this research, the core idea for enabling model-free controller design is the introduction of the SDOF VCO between the actuator and the real controlled object [24]. First, Fig. 2(a) shows a real mechanical configuration to conduct active oscillation control for an arbitrary structure. The actuator is characterized by the mechanical properties of mass m_a , stiffness k_a , and damping c_a . The purpose of this configuration is to calculate the control input u realized by the actuator to attenuate vibration x_1 (vertical displacement) of the real controlled object.

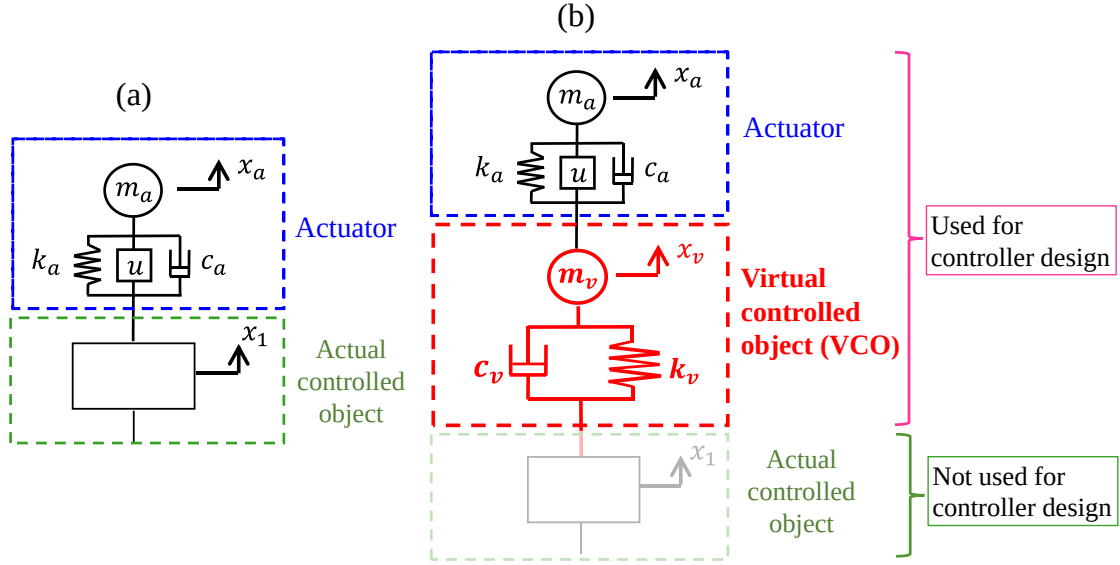


Fig. 2: VCO-based model-free controller design: (a) actual system and (b) system with VCO.

A model-free controller design is allowed by the modified system shown in Fig. 2 (b). The key strategy is to newly interpose the VCO between the actual plant (i.e., arbitrary controlled structure) and the actuator. The key idea of the VCO-MFC design is as follows: instead of using the real controlled object's model, structure only including the VCO and actuator (i.e., a 2DOF system) is utilized for design of the vibration controller. Rather than the real controlled object, the VCO with spring constant k_v , mass m_v , and damper c_v is regarded as a temporary controlled structure. Note that active damping of x_1 (vibration of the actual object) is indirectly achieved by the designed controller for VCO to damp x_v . A specific design policy for the VCO's parameters is given by the following inequality (1) to be satisfied, according to our previous research [24].

$$\frac{1}{m_v} \left\{ (\Omega_k^{cont})^2 + k_v \left(-\frac{1}{m_a} + \frac{(\Omega_k^{cont})^2}{k_a} \right) \right\} > (\Omega_k^{cont})^2 \left(-\frac{1}{m_a} + \frac{(\Omega_k^{cont})^2}{k_a} \right), \quad (1)$$

$(k = 1, 2).$

According to previous literature [24], Eq. (1) can be satisfied by properly setting k_v and m_v , resulting in $x_v \approx x_1$ in the desired frequency band. MFC design (indirect damping of x_1 via control of x_v) is then achieved. The lower and the upper bound frequencies in the frequency band are denoted as Ω_1^{cont} and Ω_2^{cont} ($\Omega_2^{cont} > \Omega_1^{cont} \geq \sqrt{k_0/m_0}$), respectively. The settings employed in this work are listed in Table 1.

Table 1. Parameters for the model-free control system design.

Properties	Value	Unit
m_a	0.2013	kg
k_a	3518	N/m
c_a	1.186	Ns/m
m_v	1.0×10^{-5}	kg
k_v	7.0×10^5	N/m
c_v	0.0	Ns/m

The MFC design needs the state-space representation of the 2DOF system in Fig. 2(b). Using the oscillation of the real controlled object, Eq. (2) defines the exoteric disturbance $w \in \mathbb{R}$ as

$$w = c_v \dot{x}_1 + k_v x_1. \quad (2)$$

Then, the state space representation is given as

$$\dot{\mathbf{x}}_{MF} = \mathbf{A}_{MF} \mathbf{x}_{MF} + \mathbf{B}_{MF1} w + \mathbf{B}_{MF2} u, \quad (3)$$

$$\mathbf{x}_{MF} = [x_v \quad x_a \quad \dot{x}_v \quad \dot{x}_a]^T, \quad (4)$$

$$\mathbf{A}_{MF} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_a + k_v}{m_v} & \frac{k_a}{m_v} & -\frac{c_a + c_v}{m_v} & \frac{c_a}{m_v} \\ \frac{k_a}{m_a} & -\frac{k_a}{m_a} & \frac{c_a}{m_a} & -\frac{c_a}{m_a} \end{bmatrix}, \quad (5)$$

$$\mathbf{B}_{MF1} = \begin{bmatrix} 0 & 0 & \frac{1}{m_v} & 0 \end{bmatrix}^T, \quad (6)$$

$$\mathbf{B}_{MF2} = \begin{bmatrix} 0 & 0 & -\frac{1}{m_v} & \frac{1}{m_a} \end{bmatrix}^T, \quad (7)$$

It should be remarked that only the specifications of the actuator and the VCO are used in each coefficient matrix of the state equation. Hence, a model-free controller can be derived by directly applying a familiar MBC theory to Eq. (3).

3. VCO-based model-free SMC

Sliding mode control (SMC) is a representative robust nonlinear control theory characterized by discontinuous switching behavior of the control input. The implementation procedure consists in two stages: (1) sliding surface design and (2) nonlinear control input design to ensure the existence of a sliding mode. A remarkable feature of our work is that a model-free SMC is easily constructed by directly applying these two procedures to the VCO-based system (3).

3.1. Design of sliding surface

It is first necessary to specify a sliding surface $\sigma = 0$, which represents the ideal system dynamics during sliding-mode. A typical form of a switching function $\sigma \in \mathbb{R}$ is as follows [61].

$$\sigma = \mathbf{S}\mathbf{x}_{MF} \quad (8)$$

where $\mathbf{S} \in \mathbb{R}^{1 \times 4}$ is the coefficient vector to be determined.

The system can be converted to the equivalent form by using variable transformation with a transformation matrix $\mathbf{T}_{tr} \in \mathbb{R}^{4 \times 4}$ as follows:

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{B}_{22} \end{bmatrix} u, \quad (9)$$

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \mathbf{T}_{tr} \mathbf{x}_{MF}, \quad (10)$$

$$\xi_1 \in \mathbb{R}^{3 \times 1}, \xi_2 \in \mathbb{R}^{1 \times 1},$$

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} = \mathbf{T}_{tr} \mathbf{A}_{MF} \mathbf{T}_{tr}^{-1}, \quad (11)$$

$$\mathbf{A}_{11} \in \mathbb{R}^{3 \times 3}, \mathbf{A}_{12} \in \mathbb{R}^{3 \times 1}, \mathbf{A}_{21} \in \mathbb{R}^{1 \times 3}, \mathbf{A}_{22} \in \mathbb{R}^{1 \times 1},$$

$$\mathbf{T}_{tr} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{B}_{21} \mathbf{B}_{22}^{-1} \\ \mathbf{0}_{1 \times 3} & \mathbf{I}_{1 \times 1} \end{bmatrix}. \quad (12)$$

According to the previous work [28], we can specify the switching function σ based on the LQR theory [61] as:

$$\sigma = \mathbf{S}\mathbf{x}_{MF} = [\mathbf{A}_{12}^T \mathbf{P}_s + \mathbf{q}_{12}^T \quad \mathbf{q}_{22}] \mathbf{T}_{tr} \mathbf{x}_{MF}. \quad (13)$$

where $\mathbf{P}_s$ is a unique positive-definite symmetric matrix computed via an algebraic Riccati equation and $\mathbf{q}_{12} \in \mathbb{R}^{3 \times 1}$ and $\mathbf{q}_{22} \in \mathbb{R}^{1 \times 1}$ are weighting factors. Their details can be found in the literature [28]. Consequently, the sliding surface $\sigma = 0$ can be defined in phase space.

3.2. Control input design and convergence analysis

The control law needs to be designed to ensure the existence condition of a sliding mode, making state trajectory converge to the sliding surface. One candidate is given as [62]:

$$\begin{aligned} u &= u_{eq} + u_{nl} \\ &= -(\mathbf{S}\mathbf{B}_{MF2})^{-1} \mathbf{S}\mathbf{A}_{MF} \mathbf{x}_{MF} - k|\sigma|^{-\alpha} \text{sgn}(\sigma) \end{aligned} \quad (14)$$

where switching gain is denoted as k and α is a positive variable in $0 < \alpha < 1$ [63].

The proper switching control gain should be specified for Eq. (14) so as to establish the reaching condition for $\sigma \rightarrow 0$. Here, the Lyapunov function approach is employed to analyze the convergence [42][61]. We can select the Lyapunov function for σ as:

$$V = \frac{1}{2} \sigma^2 > 0. \quad (15)$$

Using the time derivative of V , $\sigma \rightarrow 0$ is realized by the following condition.

$$\dot{V} = \sigma^T \dot{\sigma} < 0 \quad \text{for } \forall \sigma \neq 0. \quad (16)$$

According to the previous study [28], condition (17) is equivalent to satisfying (16).

$$\text{sgn}(\sigma)\mathbf{SB}_{MF1}w < k'. \quad (17)$$

Here,

$$k' = k|\sigma|^{-\alpha}\mathbf{SB}_{MF2}. \quad (18)$$

In VCO-MFC, it should be noted that $x_v \approx x_1$ holds in the controlled frequency band. Hence, w can be rearranged with $c_v = 0$ as:

$$w = k_v x_1 \approx k_v x_v = k_v [1 \ 0 \ 0 \ 0] \mathbf{x}_{MF}. \quad (19)$$

According to the previous study [28], the condition (17) can be evaluated using Eq. (19) as follows.

$$\text{sgn}(\sigma)\mathbf{SB}_{MF1}w \leq \|\mathbf{S}\| \frac{k_v}{m_v} \|\mathbf{x}_{MF}\|_{max}. \quad (20)$$

Here, $\|\mathbf{x}_{MF}\|_{max}$ is the upper bound for the norm of the state vector $\mathbf{x}_{MF}$. Hence, the condition to ensure the existence of a sliding mode is as follows.

$$\|\mathbf{S}\| \frac{k_v}{m_v} \|\mathbf{x}_{MF}\|_{max} < k'', \quad (21)$$

$$k = k''(\mathbf{SB}_{MF2})^{-1}|\sigma|^\alpha. \quad (22)$$

For $\sigma \rightarrow 0$, the switching gain k'' is set to satisfy the above inequality. The integration of the switching control $u_{nl} = -k''(\mathbf{SB}_{MF2})^{-1}\text{sgn}(\sigma)$ and equivalent input $u_{eq} = -(\mathbf{SB}_{MF2})^{-1}\mathbf{SA}_{MF}\mathbf{x}_{MF}$ results in the control effort:

$$u = -(\mathbf{SB}_{MF2})^{-1}\mathbf{SA}_{MF}\mathbf{x}_{MF} - k''(\mathbf{SB}_{MF2})^{-1}\text{sgn}(\sigma), \quad (23)$$

This work set k'' as 1.0×10^1 considering the inequality (21).

Instead of $\|\mathbf{x}_{MF}\|_{max}$, we evaluate $\|\mathbf{x}_{MF}\|$ estimated via a model-free Kalman filter [28], which is designed using system (3). Therefore, the above condition evaluates the estimated norm $\|\hat{\mathbf{x}}_{MF}\|$ in real time. By employing the following u_{nl} , the convergence condition is always established.

$$u_{nl} = \begin{cases} -k''(\mathbf{SB}_{MF2})^{-1}\|\hat{\mathbf{x}}_{MF}\|\text{sgn}(\sigma), & \|\mathbf{S}\| \frac{k_v}{m_v} \|\hat{\mathbf{x}}_{MF}\| \geq k'' \\ -k''(\mathbf{SB}_{MF2})^{-1}\text{sgn}(\sigma), & \|\mathbf{S}\| \frac{k_v}{m_v} \|\hat{\mathbf{x}}_{MF}\| < k'' \end{cases}, \quad (24)$$

$$\|\mathbf{S}\| \frac{k_v}{m_v} < k''. \quad (25)$$

4. VCO-based model-free fuzzy sliding mode control (VCO-MFFSMC)

4.1. Introduction of fuzzy rules

Severe chattering will be caused by the control signal (23) because of the sign function $\text{sgn}(\sigma)$ [61], [64]. From the theoretical viewpoint, because high-frequency chattering unnecessarily excites unpredictable high-frequency vibration modes (i.e., unmodeled dynamics) of a controlled structure, it leads to a deterioration of vibration control performance. The chattering also adversely affects the closed-loop stability. Theoretically, a deterioration mechanism of the performance of active vibration control is as follows; 1) chattering is originally owing to $\text{sgn}(\sigma)$ included in the control signal in Eq. (23). 2) the

sign function involves high-frequency switching of the control input (i.e., chattering effect), 3) Due to the high-frequency switching, vibration modes of a structure are excited especially in a high-frequency band. 4) In addition, because chattering forces an actuator to be driven at speeds that ignore the limits of responsiveness of an actual hardware system, the actuator cannot follow the control command signal, thus degrading system stability. That is, an actuator and other equipment are subjected to loads that cause damage.

In order to avoid the adverse effects of chattering phenomena on hardware systems and vibration control performances mentioned above, chattering mitigation is necessary.

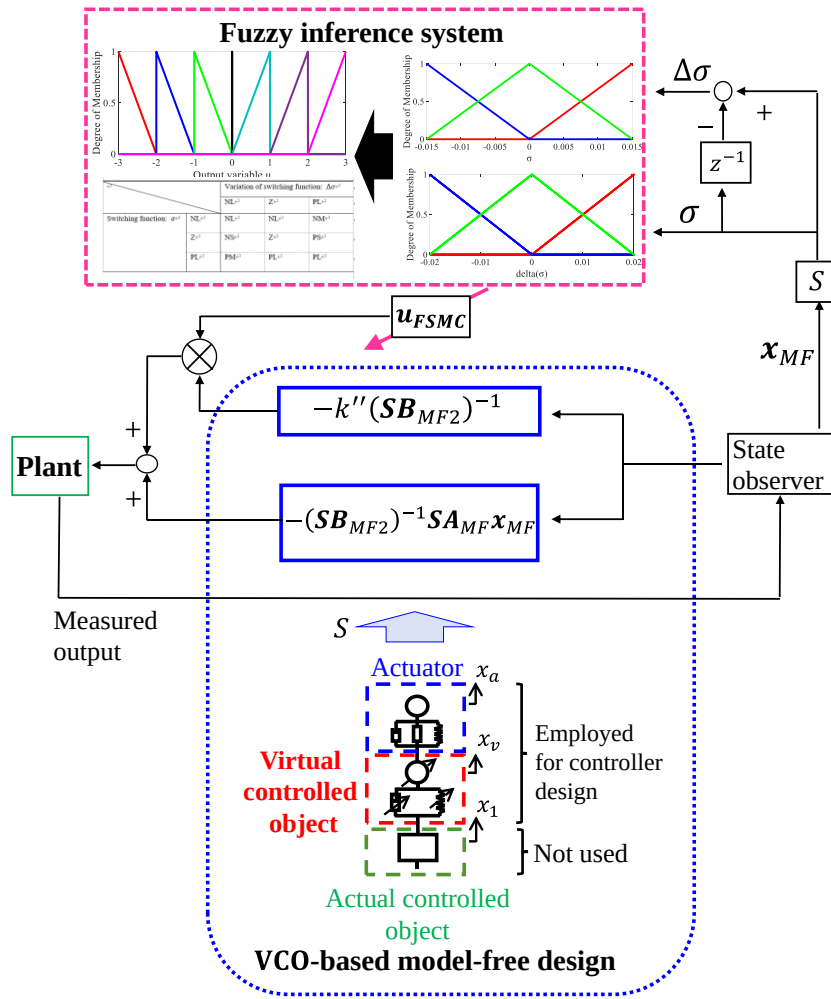


Fig. 3: Outline of the model-free fuzzy SMC.

This work constructs a fuzzy inference-based adaptive compensation system to address the chattering issue. Figure 3 shows the outline of the proposed fuzzy SMC, namely VCO-MFFSMC. The aim of the fuzzy inference is to prevent intensive switching of the control input, which causes the chattering, while ensuring that a system state trajectory rapidly converges to the sliding surface ($\sigma = 0$). For adjusting the

control input values, the rate of change of σ , denoted as $\Delta\sigma$, is therefore taken into account, unlike standard SMCs. In the fuzzy inference, the input linguistic variables are σ and $\Delta\sigma$, and the output linguistic variable is the control input effort u_f . In this study, the fuzzy rules are determined based on the strategy presented in the previous paper [65] as the effective principle for chattering reduction. Specifically, the fuzzy rules are designed so as to realize the following compensation strategy to always ensure $\sigma \rightarrow 0$. A large control effort is applied to make the state trajectory return to $\sigma = 0$ when σ is drastically departing from $\sigma = 0$ and $\Delta\sigma$ also promotes this departure. In contrast, we need to gradually decrease a control effort when σ is naturally approaching to $\sigma = 0$. When the state trajectory is near $\sigma = 0$, a control input needs to be small. According to the previous study [65], the nine fuzzy rules are employed in order to always ensure $\sigma \rightarrow 0$ as follows:

(Rule 1).

IF (σ is **Negative Large (NL)**) and ($\Delta\sigma$ is **Negative Large (NL)**) THEN (u_f is **Negative Large (NL)**), (26)

(Rule 2).

IF (σ is **Negative Large (NL)**) and ($\Delta\sigma$ is **Zero (Z)**) THEN (u_f is **Negative Large (NL)**), (27)

(Rule 3).

IF (σ is **Negative Large (NL)**) and ($\Delta\sigma$ is **Positive Large (PL)**) THEN (u_f is **Negative Medium (NM)**), (28)

(Rule 4).

IF (σ is **Zero (Z)**) and ($\Delta\sigma$ is **Negative Large (NL)**) THEN (u_f is **Negative Small (NS)**), (29)

(Rule 5).

IF (σ is **Zero (Z)**) and ($\Delta\sigma$ is **Zero (Z)**) THEN (u_f is **Zero (Z)**), (30)

(Rule 6).

IF (σ is **Zero (Z)**) and ($\Delta\sigma$ is **Positive Large (PL)**) THEN (u_f is **Positive Small (PS)**), (31)

(Rule 7).

IF (σ is **Positive Large (PL)**) and ($\Delta\sigma$ is **Negative Large (NL)**) THEN (u_f is **Positive Medium (PM)**), (32)

(Rule 8).

IF (σ is **Positive Large (PL)**) and ($\Delta\sigma$ is **Zero (Z)**) THEN (u_f is **Positive Large (PL)**), (33)

(Rule 9).

IF (σ is **Positive Large (PL)**) and ($\Delta\sigma$ is **Positive Large (PL)**) THEN (u_f is **Positive Large (PL)**). (34)

Table 2 summarizes the above fuzzy rules [65].

Table 2. Fuzzy rule table of Rules 1-9 in (47)-(55)

		Variation of switching function: $\Delta\sigma$		
		NL	Z	PL
Switching function: σ	NL	NL	NL	NM
	Z	NS	Z	PS
	PL	PM	PL	PL

4.2. Fuzzy inference by Tsukamoto method

In this study, the Tsukamoto-type inference method [66][67][68] is basically used because of its computational simplicity. Figures 4(a), (b), and (c) show the membership functions of the fuzzy sets with respect to σ , $\Delta\sigma$ and u_f , respectively. For u_f in the consequent part, the right-angled triangular membership functions as well as the fuzzy singleton (i.e., a crisp value “0”), which particularly represents “**Zero (Z)**”, are used in this study. Therefore, the inference process can be formulated as a Tsukamoto-type method that partially uses the fuzzy singleton, reducing the online computational requirements. The membership functions of the fuzzy sets {**NL**, **NM**, **NS**, **Z**, **PS**, **PM**, **PL**} of u_f are as follows:

$$h_{NL}^u(u_f) = \begin{cases} -\frac{1}{a}(u_f + 2a), & (-3a \leq u_f \leq -2a), \\ 0, & \text{otherwise} \end{cases} \quad (35)$$

$$h_{NM}^u(u_f) = \begin{cases} -\frac{1}{a}(u_f + a), & (-2a \leq u_f \leq -a), \\ 0, & \text{otherwise} \end{cases} \quad (36)$$

$$h_{NS}^u(u_f) = \begin{cases} -\frac{1}{a}u_f, & (-a \leq u_f \leq 0), \\ 0, & \text{otherwise} \end{cases} \quad (37)$$

$$h_Z^u(u_f) = \begin{cases} 1, & (u_f = 0) \\ 0, & \text{otherwise} \end{cases} \quad (38)$$

$$h_{PS}^u(u_f) = \begin{cases} \frac{1}{a}u_f, & (0 \leq u_f \leq a), \\ 0, & \text{otherwise} \end{cases} \quad (39)$$

$$h_{PM}^u(u_f) = \begin{cases} \frac{1}{a}(u_f - a), & (a \leq u_f \leq 2a), \\ 0, & \text{otherwise} \end{cases} \quad (40)$$

$$h_{PL}^u(u_f) = \begin{cases} \frac{1}{a}(u_f - 2a), & (2a \leq u_f \leq 3a) \\ 0, & \text{otherwise} \end{cases} \quad (41)$$

where $a = u_{max}/3$ is a tuning variable to be specified.

Based on Tsukamoto-type reasoning [66][67][68], the inference result u_i ($i = 1 - 9$) from each rule is calculated by substituting the firing strength of each rule $0 \leq w_i \leq 1$ ($i = 1 - 9$) into the inverse functions of the membership functions of the fuzzy sets of u_f as:

$$u_1 = h_{NL}^{u-1}(w_1) = -aw_1 - 2a \quad (-3a \leq u_1 \leq -2a), \quad (42)$$

$$u_2 = h_{NL}^{u-1}(w_2) = -aw_2 - 2a \quad (-3a \leq u_2 \leq -2a), \quad (43)$$

$$u_3 = h_{NM}^{u-1}(w_3) = -aw_3 - a \quad (-2a \leq u_3 \leq -a), \quad (44)$$

$$u_4 = h_{NS}^{u-1}(w_4) = -aw_4 \quad (-a \leq u_4 \leq 0), \quad (45)$$

$$u_5 = h_Z^{u-1}(w_5) = 0 \quad (u_5 = 0), \quad (46)$$

$$u_6 = h_{PS}^{u-1}(w_6) = aw_6 \quad (0 \leq u_6 \leq a), \quad (47)$$

$$u_7 = h_{PM}^{u-1}(w_7) = aw_7 + a \quad (a \leq u_7 \leq 2a), \quad (48)$$

$$u_8 = h_{PL}^{u-1}(w_8) = aw_8 + 2a \quad (2a \leq u_8 \leq 3a), \quad (49)$$

$$u_9 = h_{PL}^{u-1}(w_9) = aw_9 + 2a \quad (2a \leq u_9 \leq 3a). \quad (50)$$

Considering all of Rules 1-9, the resultant inference result, which is used as fuzzy control input u_{FSMC} , can be derived as a weighted average of u_i by w_i as:

$$u_{FSMC} = \frac{\sum_{i=1}^9 w_i u_i}{\sum_{i=1}^9 w_i}. \quad (51)$$

To construct VCO-MFFSMC, $\text{sgn}(\sigma)$ in Eq. (23) is replaced with u_{FSMC} as:

$$u = -(\mathbf{SB}_{MF2})^{-1} \mathbf{SA}_{MF} \mathbf{x}_{MF} - k'' (\mathbf{SB}_{MF2})^{-1} u_{FSMC}. \quad (52)$$

According to Eq. (52), the chattering phenomena can be avoided by replacing $\text{sgn}(\sigma)$ (i.e., the source of discontinuous changes of the control signal) with u_{FSMC} . The introduction of $\Delta\sigma$ for adjusting the control effort amount also helps to reduce chattering, unlike the standard SMC. The specifications employed for VCO-MFFSMC are shown in Table 3.

Table 3. Parameters for the control system design

Properties	Value
k''	1.0×10^1
u_{max}	3.0 N
a	1.0 N

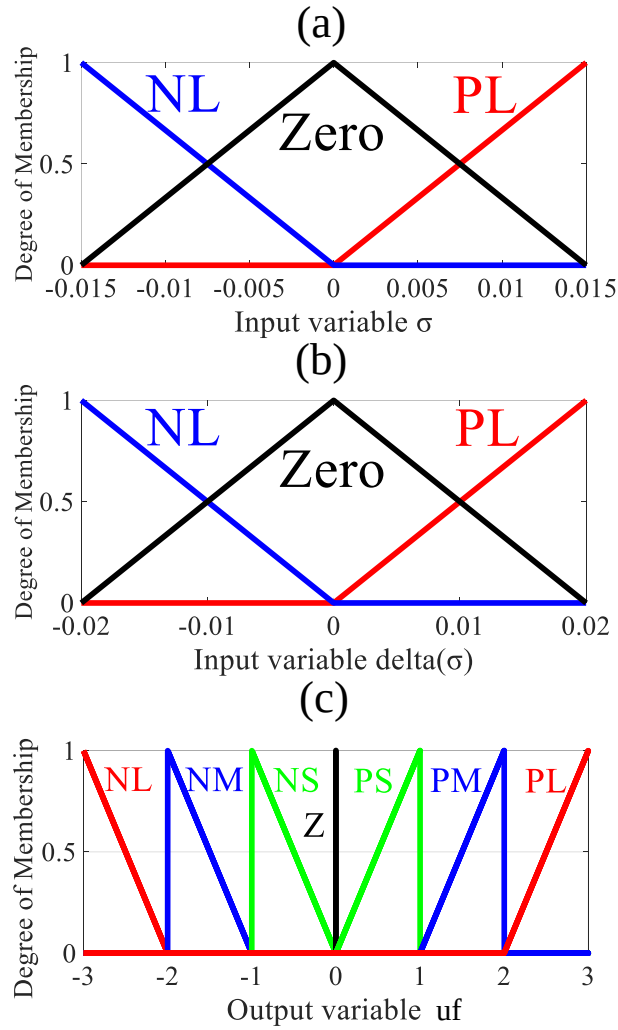


Fig. 4: Membership functions of the fuzzy sets: (a) switching function, (b) change of the switching function, and (c) control input.

Focusing on the membership functions shown in Fig. 4 (c), which more directly affect the control performance because they are placed in the output parts of the fuzzy rules, the Tsukamoto-type inference method especially requires monotone decreasing or increasing shapes (e.g., right angled triangles) in a certain range. This is according to its inference mechanism [66][67][68]. From the qualitative viewpoint, larger values of $a = u_{max}/3$ of “PL” and NL” may more aggressively improve the vibration control performance because an amount of the computed control input can be increased. Nevertheless, it also should be considered that this tuning policy sometimes may sacrifice the robustness to changes in controlled structures. In addition, the firing strength of each rule $0 \leq w_i \leq 1$ in the input parts of the fuzzy rules should be higher in order to strengthen chattering mitigation effects. Hence, shaping the gradient of the membership functions “PL” and “NL” shown in Figs. 4 (a) and (b) more steeply may improve the chattering.

The quantitative viewpoints of selections of the membership functions are as follows: if the ranges over which σ and $\Delta\sigma$ vary are quantitatively identified in advance according to Eq. (8), they can be adopted as the input ranges (i.e., maximum and minimum values of the input variables) on the horizontal axis of the membership functions shown in Figs. 4 (a) and (b). Similarly, we can quantitatively know a limit value of the control input, which can be actually realized by an electromagnetic proof-mass actuator and a current amplifier used in controller implementation; therefore, the range (i.e., $a = u_{max}/3$) on the horizontal axis of the membership functions of the control input effort u_f needs to be determined considering the limitation.

5. Experimental verification

5.1. Experimental settings

The proposed approach, namely VCO-MFFSMC, is compared to the previous approach, which lacks fuzzy-inference-based chattering compensation and simply uses a smoothing function for chattering elimination. Note that the previous approach also uses the model-free SMC proposed in the previous literature [28].

The damping performance is investigated via vibration control experiments. The closed-loop system configuration and the experimental setup are shown in Fig. 5. The signal generator was used to deliver a 1–1000 Hz linear sweep sinusoidal signal to the controlled objects as the disturbance. The disturbance is applied to the structure via the shaker to excite vibrations. The vibration suppression performance of the controllers was assessed by the acceleration response of the controlled objects. The frequency responses from the disturbance to the acceleration of the damping point can be measured with the spectrum analyzer using signals from the load cell and accelerometer. These signals are amplified via two amplifiers. The accelerometer was mounted by a magnet; the actuator was fixed by bolts and nuts on the structure's damping point in the experimental system. On the rear side of the actuator mounting location, the sensor was attached. The accelerometer detected the measured output, which is then fed to the digital signal processor (DSP). Based on the detected output, the DSP computed the signal for the control input. When the measured output is fed back to the controller (i.e., DSP), a high-pass filter that has a cut off frequency of 10 Hz is applied to prevent negative effects due to zero drift of the used acceleration sensor for the control system. To avoid spillover, the DSP's control input signal was filtered through an analog low-pass filter. The actuator was then driven by the current amplifier's amplification of the control signal. The control frequency (sampling frequency) of the DSP was set as 20 kHz.

As shown in Fig. 6 (a), this study employs a cantilever plate made of aluminum as the real controlled object, referred to as *Structure 1*. In addition, to verify the robustness of the controllers, other controlled objects: *Structures 2* and *3* are prepared with dynamic characteristic changes by placing a 0.685 kg mass and a 1.37 kg mass on the cantilever plate, respectively. They are depicted in Figs. 6 (b) and (c). Therefore, the controller is tested for the three controlled objects shown in Figs. 6(a)-(c). Figure 6 (a) also indicates the positions at which the disturbance to excite the structure and control input to actively attenuate

vibrations are applied. The positions of the disturbance (excitation) and control input (actuator placement) are indicated by point A ($x = 50$ mm, $y = 10$ mm, $z = 10$ mm) and point B ($x = 198$ mm, $y = 50$ mm, $z = 10$ mm), respectively. On the opposite plane, the acceleration in the z-axis direction at the same position as the actuator is measured as the observed output.

Note that all experiments consistently employ one same controller of VCO-MFFSMC. This is aimed to check robustness to characteristic variations occurring in the controlled object.

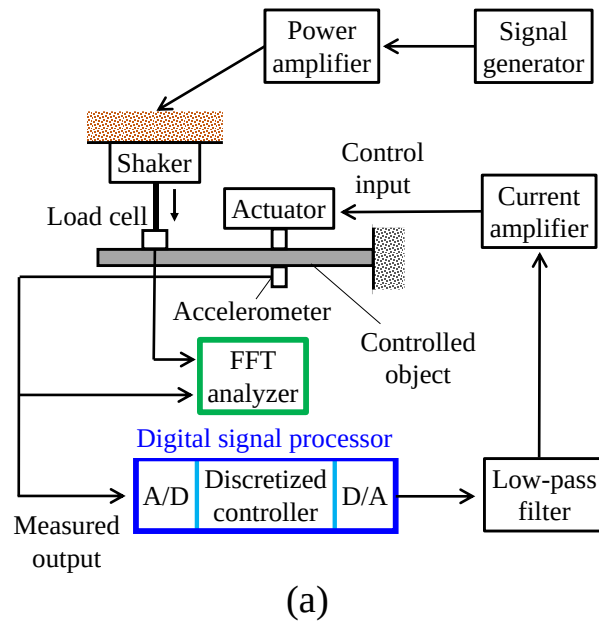
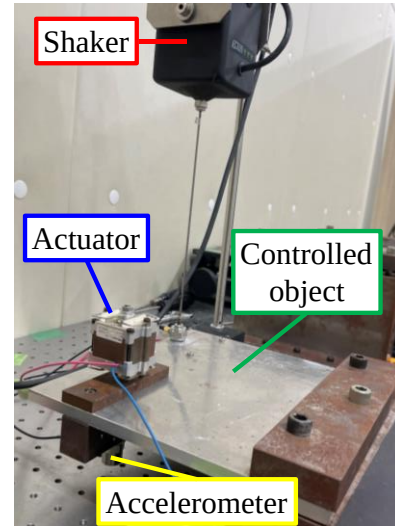


Fig. 5: Experimental system: (a) closed-loop system configuration and (b) real setup.

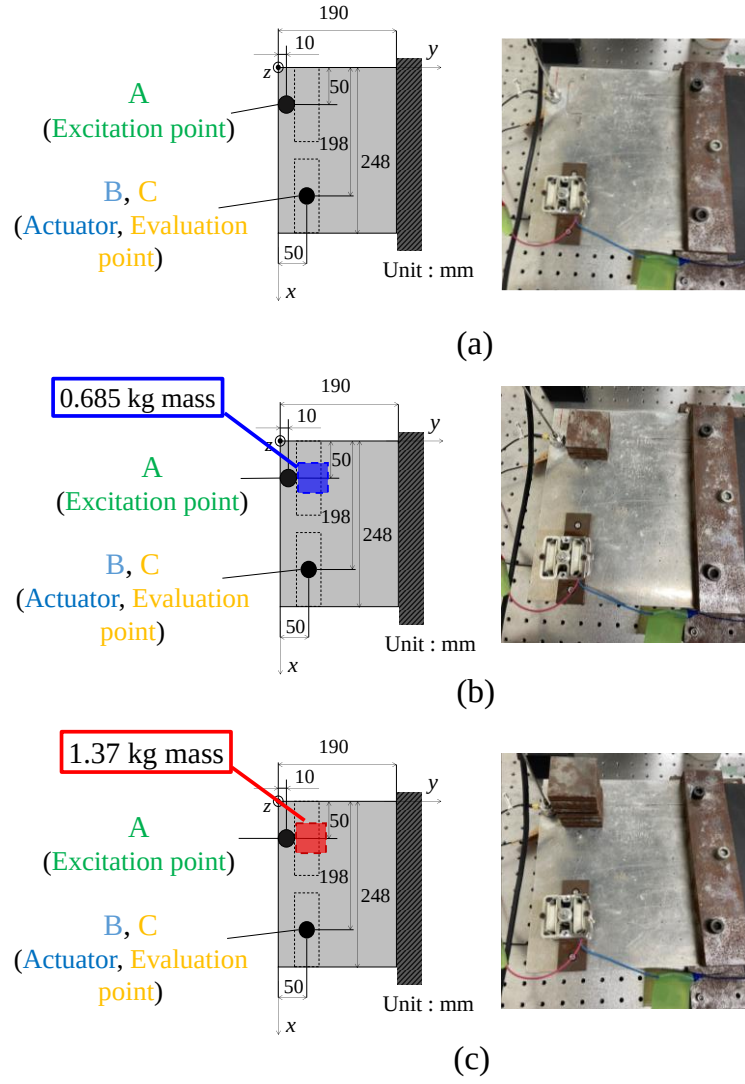


Fig. 6: Controlled objects: (a) *Structure 1*, (b) *Structure 2*, and (c) *Structure 3*.

5.2. Experimental results

The frequency responses from the disturbance (load cell output) to the measured acceleration (sensor output), for *Structure 1*, 2, and 3, are shown in Figs. 7, 8, and 9, respectively. All experiments employed the same controller for each of VCO-MFFSMC and the previous approach. For each graph, the open-loop response without control, closed-loop frequency response with the previous approach, and response with the proposed approach (i.e., VCO-MFFSMC) are denoted in blue, green, and red lines, respectively.

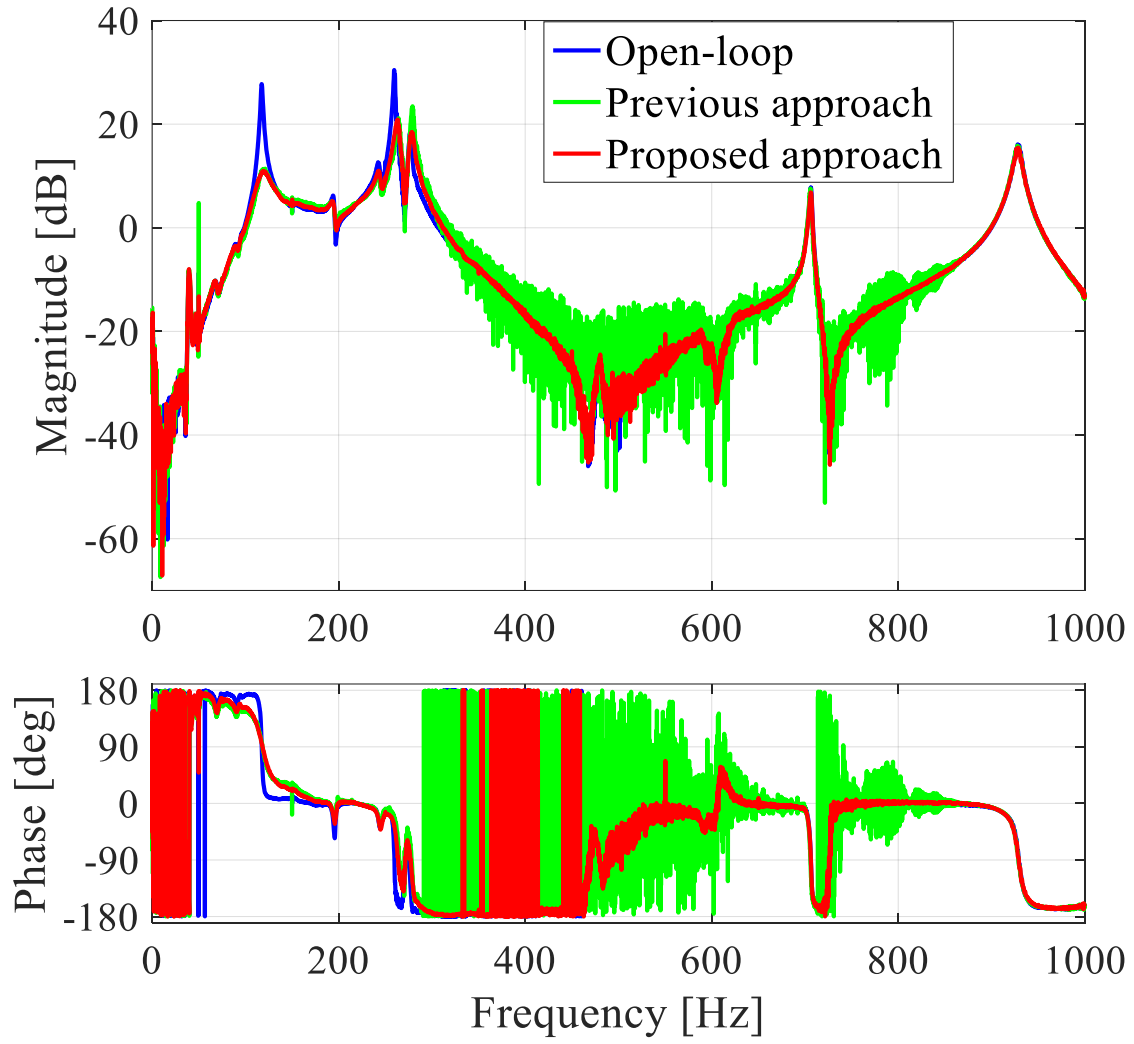


Fig. 7: Frequency responses from the disturbance (load cell output) to the measured acceleration (sensor output) for *Structure 1*.

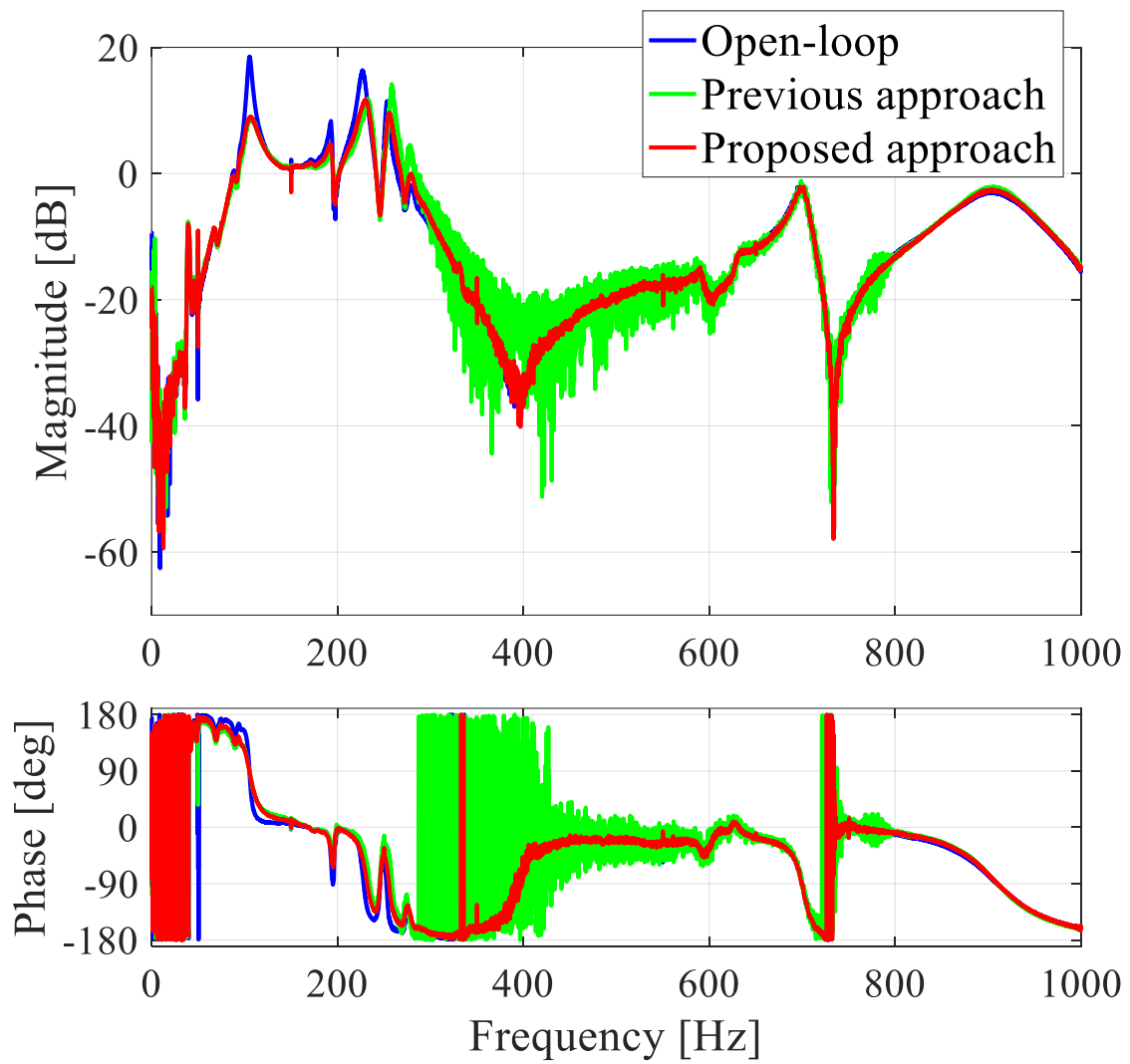


Fig. 8: Frequency responses from the disturbance to the measured acceleration for *Structure 2*.

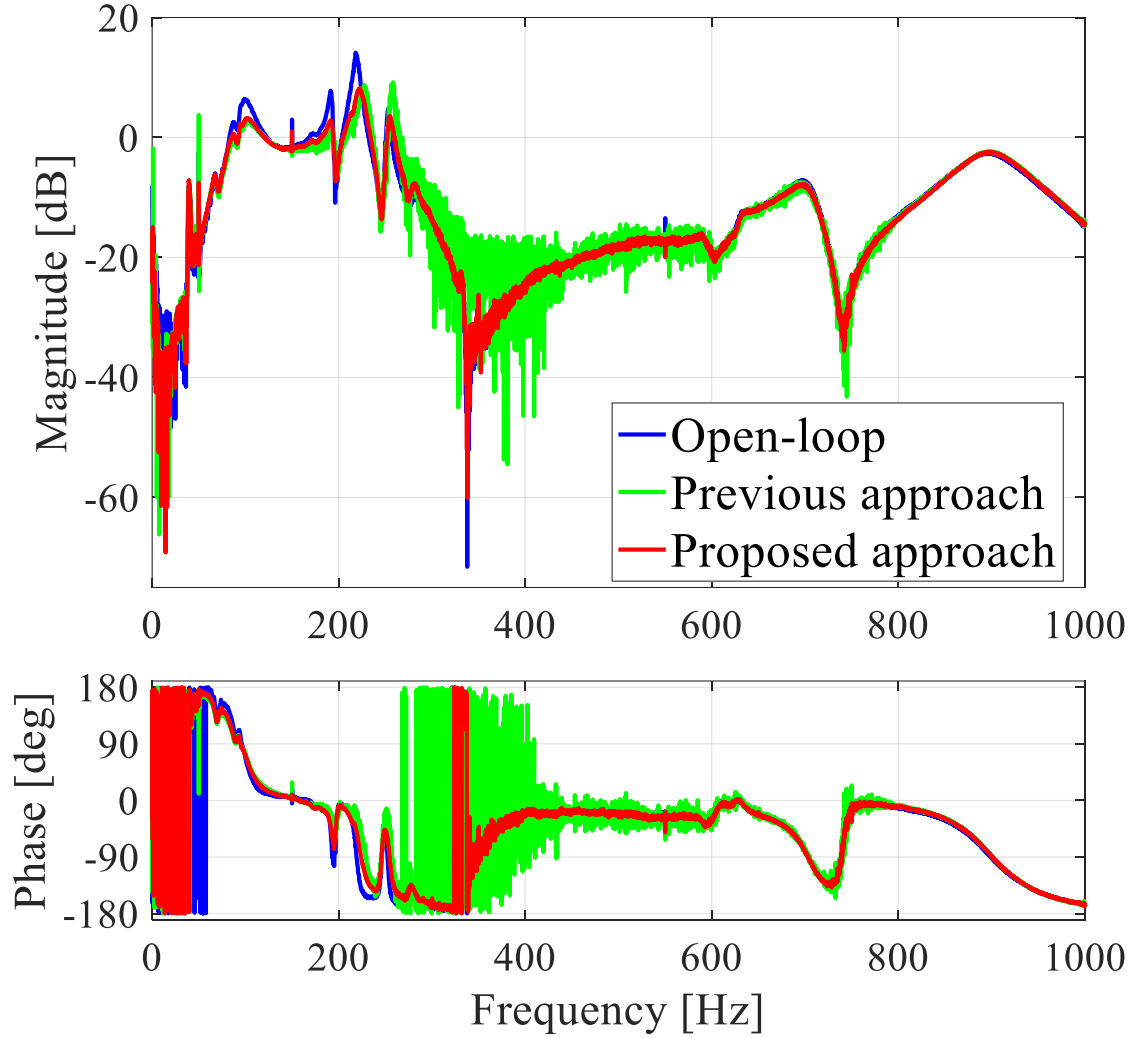


Fig. 9: Frequency responses from the disturbance to the measured acceleration for *Structure 3*.

5.3. Discussion

For the main resonance peaks in the low frequency band below 300 Hz in Figs. 7, 8, and 9, the proposed VCO-MFFSMC provides sufficient damping effects, in contrast to the open-loop response. In addition, the previous approach also reduces the resonance peaks at almost same level as VCO-MFFSMC. Tables 4, 5, and 6 summarize the approximate representative damping effects in Figs. 7, 8, and 9, respectively. These results clearly indicate the effectiveness of the VCO-based model-free design, which relies on the key condition $T_{xvx1} \approx 1$ (i.e., $x_v \approx x_1$) in the controlled frequency band. Comparison of the red and green lines below 300 Hz confirms that the convergence property of SMC, which was presented in the previous literature [28], can be also obtained even though the sign function is replaced with the fuzzy control input u_{FSMC} . Namely, the fuzzy rules employed in this paper well work in the framework of VCO-MFC. The effectiveness of the similar fuzzy rules has been already discussed [65]; however, it should be remarked that this paper formulates the inference through a Tsukamoto-type method, which

decreases the online computational effort.

Table 4. Approximate main peak reduction for *Structure 1*.

Controller	Reduction amount [dB]	
	117.5 Hz peak in open-loop response	259.5 Hz peak in open-loop response
VCO-MFFSMC	16.8000	9.5600
Previous approach	16.4200	9.3000

Table 5. Approximate main peak reduction for *Structure 2*.

Controller	Reduction amount [dB]	
	105.3 Hz peak in open-loop response	227 Hz peak in open-loop response
VCO-MFFSMC	9.5350	4.6700
Previous approach	9.6610	4.5200

Table 6. Approximate main peak reduction for *Structure 3*.

Controller	Reduction amount [dB]	
	99.22 Hz peak in open-loop response	218.3 Hz peak in open-loop response
VCO-MFFSMC	3.1120	5.9500
Previous approach	3.5340	5.3720

As clearly seen in all Figs. 7-9, the previous approach suffers from the remaining issue, i.e., severe chattering. The green lines show that this chattering phenomenon manifests itself in the form of severe waveform turbulence above 300 Hz. Considering that chattering occurs in all results regardless of the amount of characteristic variation in the controlled objects, this may be due to delays of the amplifier, signal processing, or actuator response, etc. In addition, there is a possibility that the fixed parameters of the previous SMC, e.g., switching gain and variables for chattering reduction, are not optimal. In summary, the green lines indicate that use of only smoothing functions [28], [43] is insufficient to eliminate chattering.

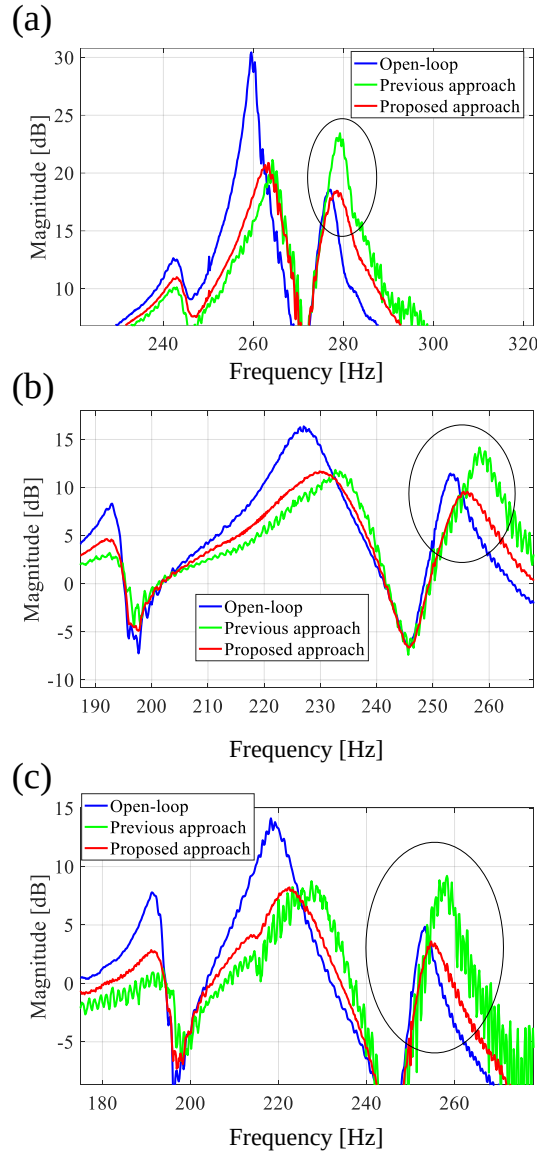


Fig. 10: Enlarged views of the magnitude of the frequency response: (a) *Structure 1*, (b) *Structure 2*, and (c) *Structure 3*.

From the experimental viewpoint, as clearly indicated by the green lines in Figs. 7-9, the chattering effect can be observed in the form of the disturbances in the waveforms of the frequency responses. In other words, the performance of active control is significantly deteriorated by severe chattering, which excites high-frequency vibration modes of the structures. Figure 10 shows enlarged views of the magnitude of the frequency responses of (a) *Structure 1*, (b) *Structure 2*, and (c) *Structure 3*. As shown in the green lines, the resonance peaks by the previous approach are increased compared to the open-loop responses. These deteriorations are also quantitatively summarized in Table 7 and considered to be due to the chattering effects. It is because the previous approach lacks sufficient compensation for chattering.

Table 7. Deterioration of the resonance peak by the previous approach compared to the open-loop response.

Controller	Approximate deterioration amount of the resonance peak [dB] from the open-loop response		
	279 Hz peak in <i>Structure 1</i>	258 Hz peak in <i>Structure 2</i>	258 Hz peak in <i>Structure 3</i>
Previous approach	4.88	2.71	4.355

On the other hand, chattering is significantly mitigated in the results by the proposed approach. In Figs. 7-9, compared to the green line, the red line clearly shows that waveform turbulence is almost eliminated, meaning chattering reduction. These comparison results show that the employed fuzzy rules are successful in suppressing chattering without degrading the robustness and convergence of SMC. The effectiveness of fuzzy logic for chattering reduction has been confirmed in some literatures [58], [65]; however, this study experimentally demonstrates that it is true for VCO-MFC as well.

Both the red and green lines exhibit good damping effects below 300 Hz for all three controlled objects, even though considerable characteristic variations (e.g., shifts of natural frequencies and fluctuations of the vibration modes) exist among *Structure 1*, *2* and *3*. From this fact, the robustness of VCO-MFC can be discussed. The robustness is a result of the design process of VCO-MFC, which does not rely on any concrete models or parameters of particular controlled objects. All vibrations occurring at the damping point can be the target of control independently of changes in controlled structures as long as $x_v \approx x_1$ is established within the controlled frequency band below 1000 Hz.

5.4. Robustness test for various applied conditions

In this section, additional numerical experiments are performed in order to further validate the robustness of the proposed approach (VCO-MFFSMC) under various applied scenarios. The following scenarios are considered; 1) a controlled object with different geometric size, 2) test under different excitations, 3) analysis of a same structure when excited at different positions.

In order to flexibly and easily set the above various scenarios for one structure, this section performs numerical experiments focusing on another controlled structure. As illustrated in Fig. 11(a), this another controlled object is a cantilever plate [69], which is composed of a flat plate (200 mm×300 mm×10 mm) and weights (120 mm×30 mm×20mm), referred to as *Structure 4*. Figure 11(b) shows the finite element (FE) model constructed via finite element analysis software ANSYS. This modeling is merely introduced for performing numerical experiments, i.e., the model is not used for design of the active vibration controller at all. More details on this cantilever plate (*Structure 4*) such as conditions of FE analysis and physical properties can be found in the literature [69].

The structure is similar to the configuration of *Structure 1* in Fig. 6; however, the geometric size is

considerably changed in contrast to *Structure 1*, as clearly indicated in Fig. 11. In addition to the size, we also change positions at which a disturbance to excite the structure and control input to actively damp vibrations are applied. In Fig. 11(a), the positions of the control input (actuator placement) and disturbance (excitation) are indicated by magenta ($x = 155$ mm, $y = 50$ mm, $z = 10$ mm) and red ($x = 40$ mm, $y = 15$ mm, $z = 10$ mm) points, respectively. On the opposite plane, the acceleration in the z -axis direction at the same position as the actuator is measured as the observed output.

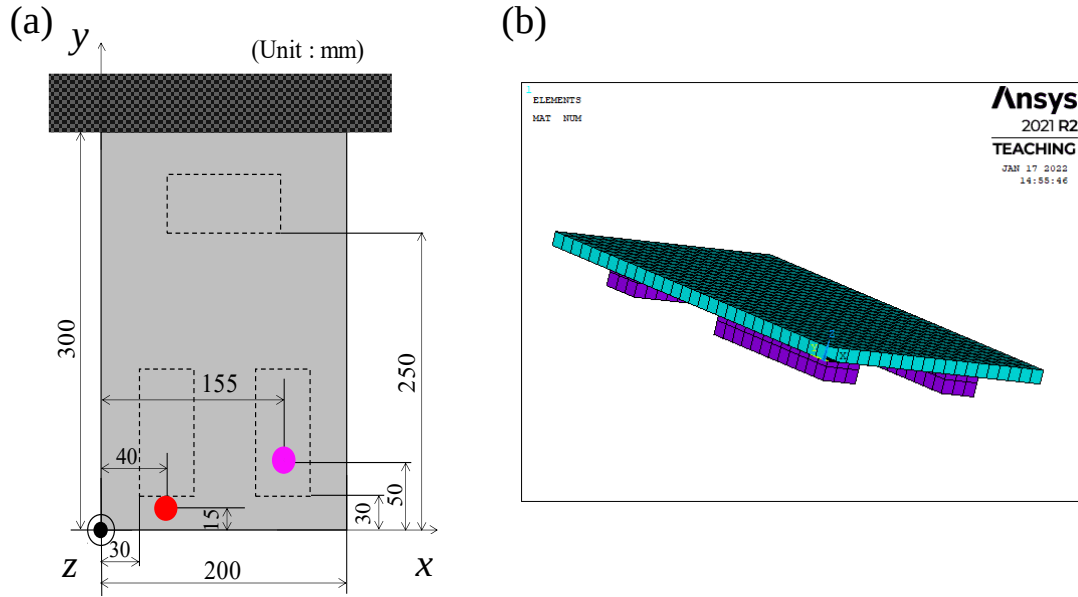


Fig. 11: Controlled object with different geometric size: (a) a cantilever plate referred to as *Structure 4* and (b) finite element (FE) model [69].

Figure 12 shows the frequency responses from the disturbance to the acceleration for *Structure 4* when the disturbance is a 1–1000 Hz linear sweep sinusoidal signal. Each of the color lines has the same meaning as that in the previous section. Corresponding to the results in the Fig. 12, Figure 13 also represents the time responses of the acceleration and control input by each of the control methods for *Structure 4*. While the lower graph indicates the time waveforms of the control inputs, the upper graph describes the time responses of the acceleration.

As demonstrated in Fig. 12, the first two resonance peaks are sufficiently suppressed by the proposed approach (VCO-MFFSMC), indicating the robustness to changes in the controlled structures such as different geometric size. It should be remarked that the same controller is consistently used in all of the tests among *Structures 1, 2, 3, and 4*. The robustness of the proposed approach to changes in the geometric size is owing to the foundation of VCO-MFC, which does not rely on any specific structures of applied objects.

In Fig. 12, the comparison of the proposed and previous approaches shows that both of their performances arrive at almost the same level. However, the time-domain analysis of Fig. 13 clearly

indicates the critical superiority of the proposed approach over the previous one, i.e., chattering mitigation. In the green line, the large chattering, which stems from the high-frequency switching of the control input, remains in both of the acceleration response and control input. This is due to the lack of adequate compensation for chattering. On the other hand, the chattering is mostly eliminated in the red line, showing that the Tsukamoto-type fuzzy inference based on the nine fuzzy rules works effectively for chattering mitigation.

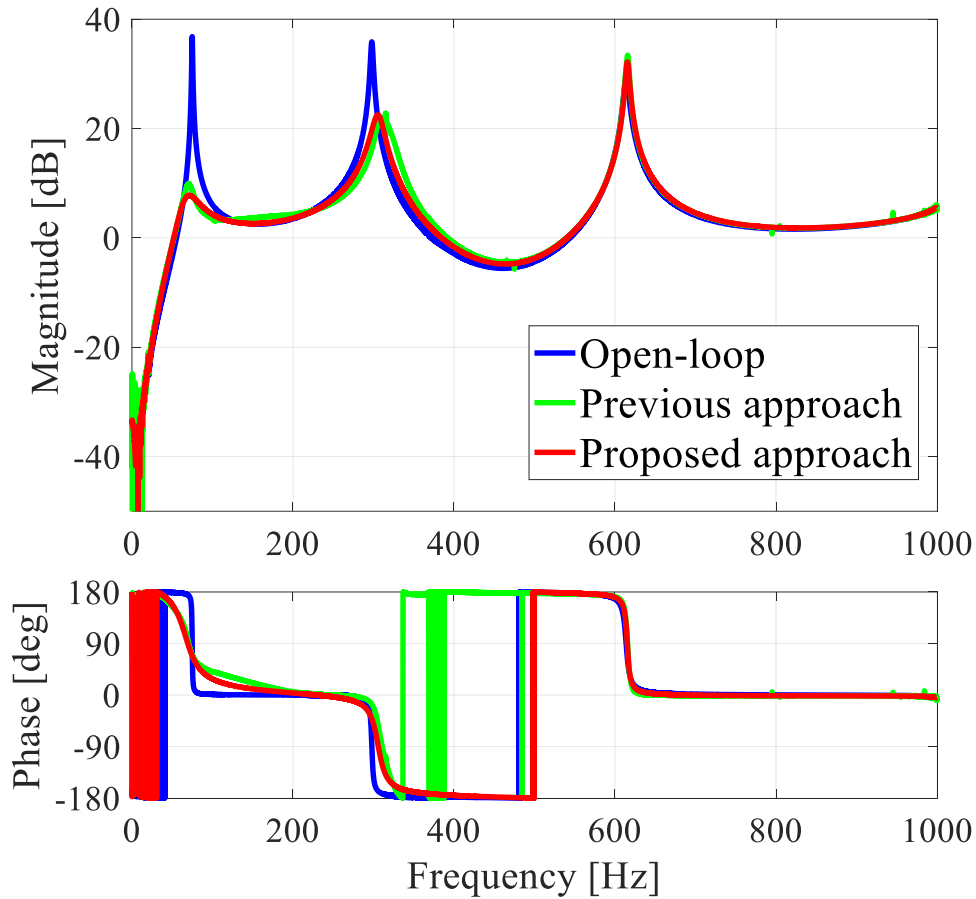


Fig. 12: Frequency responses from the disturbance to the acceleration for *Structure 4* when the disturbance is a 1–1000 Hz linear sweep sinusoidal signal.

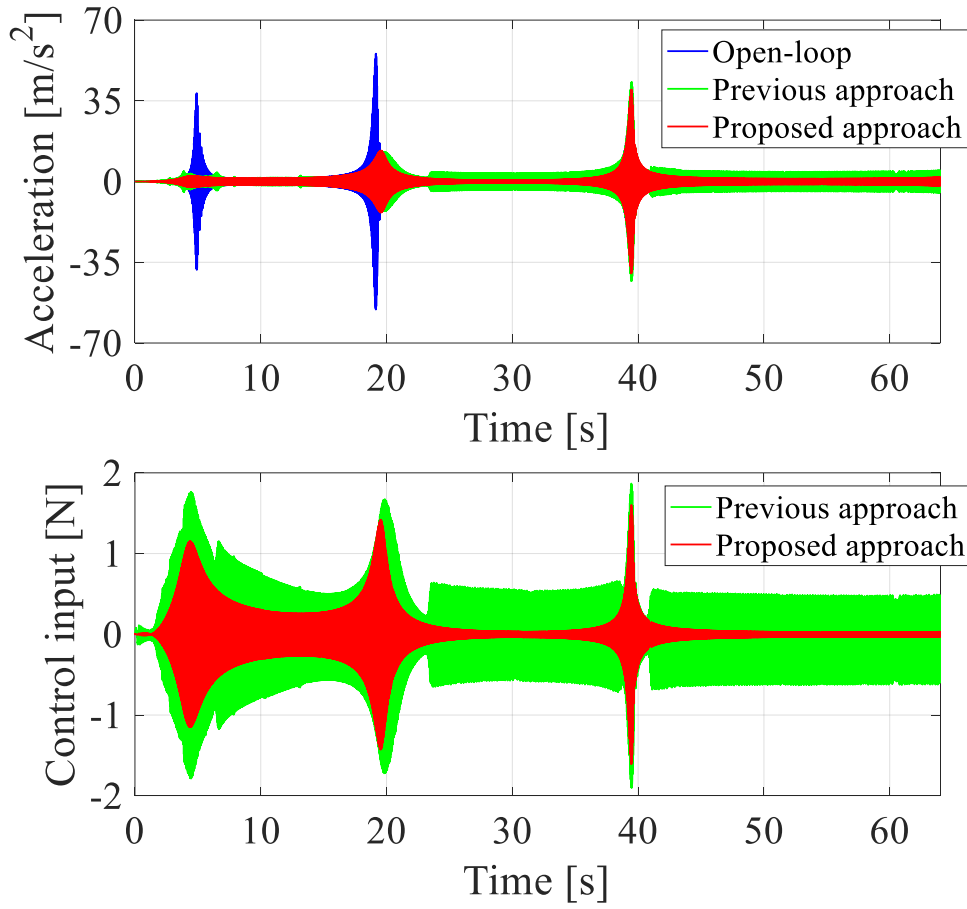


Fig. 13: Time responses of the acceleration and control input of *Structure 4* when the disturbance is a 1–1000 Hz linear sweep sinusoidal signal.

Table 8. 2-norm of the acceleration and control input when the disturbance is a 1–1000 Hz linear sweep sinusoidal signal in Fig. 13.

Controller	2-norm of the acceleration	2-norm of the control input
VCO-MFFSMC	1.2016×10^3	100.3735
Previous approach	1.4501×10^3	186.5827
Open-loop response	1.6862×10^3	—

For the time responses of the acceleration and control input shown in Fig. 13, Table 8 indicates their 2-norm to evaluate the control performance and compare the actuator power under different control methods quantitatively. In the 2-norm of the acceleration, the minimum value of VCO-MFFSMC confirms that the best damping effect is provided by the proposed approach. In addition, the comparison of the 2-norm values of the control inputs has a more critical implication on improvement by the proposed method. The reduction in the norm value of the control input by VCO-MFFSMC is owing to the effective chattering elimination, which can be also observed from the time waveform of the control input in Fig. 13. The proposed approach reduces not only the amount of the control input, but also its high-frequency

switching. That is, more actuator power can be saved in the proposed approach compared to the previous one, avoiding damage to a hardware system.

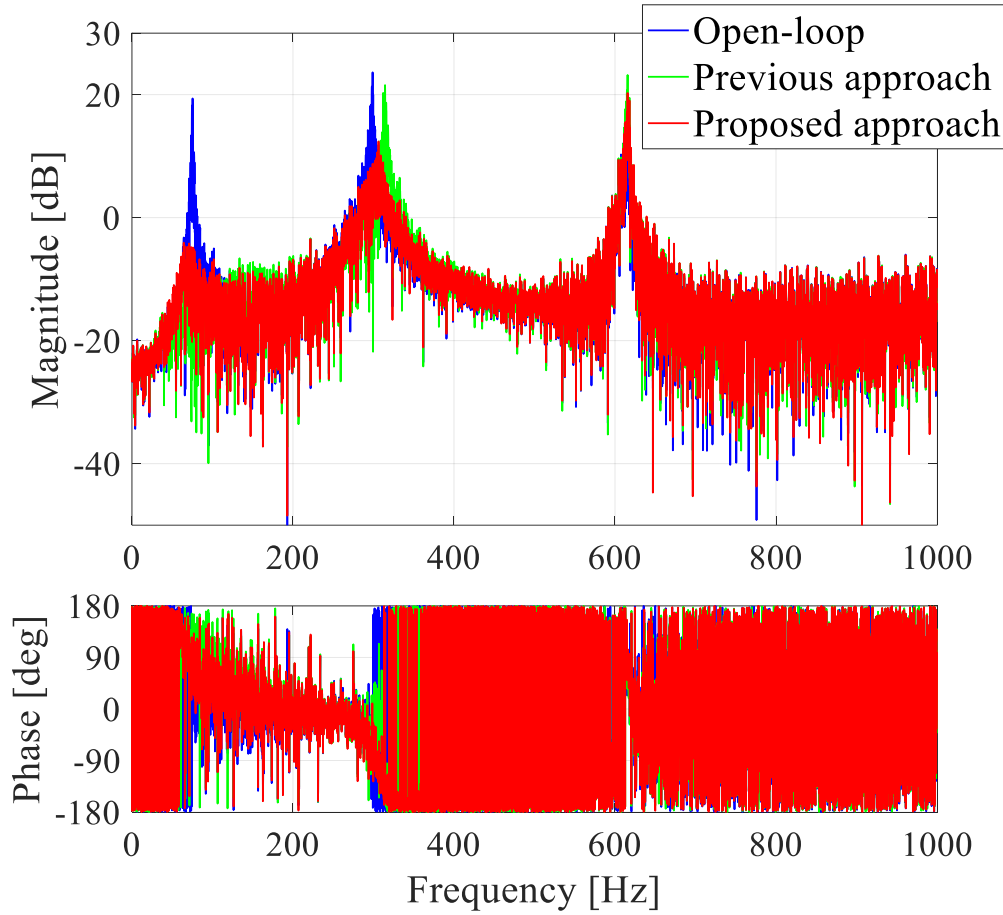


Fig. 14: Frequency responses from the disturbance to the acceleration for *Structure 4* when the disturbance is a random noise signal.

Next, an application scenario under a different type of excitation is discussed. Figures 14 and 15 represent the frequency and time responses of the control results, respectively, when the type of the disturbance is changed. Unlike Figs. 12 and 13, a random noise signal is employed for excitation in Figs. 14 and 15. In Fig. 14, the proposed approach reduces the first two resonance peaks; on the other hand, the previous approach fails to sufficiently dampen the second one. Hence, the excellent robustness of the proposed approach is confirmed even under a different excitation scenario. In Fig. 15, it should be remarked that the amount of the control input of the proposed approach is greatly decreased compared to the previous approach. In the green line, the fact that the high-frequency switching of the control input remains at the large amplitudes indicates that extra energy was expended on control. Meanwhile, it can be expected to achieve a significant reduction in control energy (i.e., actuator power) with the application of the proposed approach, as implied by the suppressed red line.

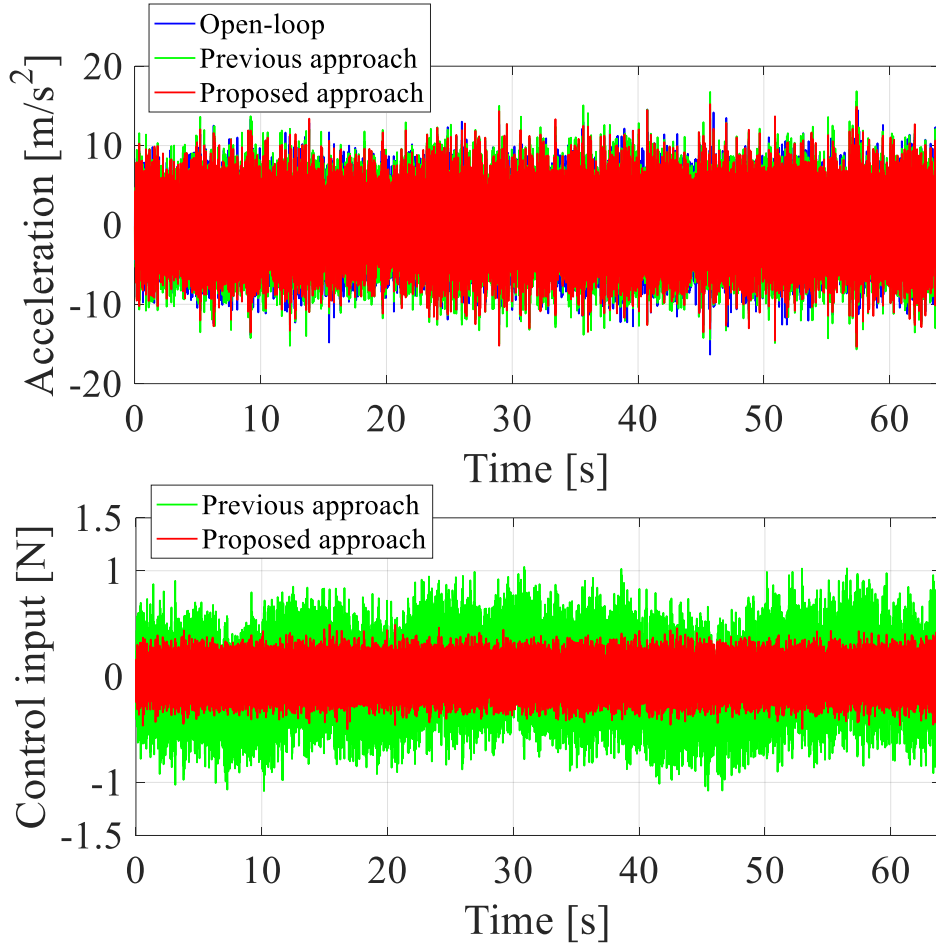


Fig. 15: Time responses of the acceleration and control input of *Structure 4* when the disturbance is a random noise signal.

For the same structure (*Structure 4*), an application scenario when excited at a different position is also considered. Figure 16 shows the frequency responses for *Structure 4* when the disturbance (a 1–1000 Hz linear sweep sinusoidal signal) is applied at a different position. Compared to the original excitation position ($x = 40$ mm, $y = 15$ mm, $z = 10$ mm) indicated in Fig. 11(a), the excitation position is changed to a different position located at ($x = 155$ mm, $y = 250$ mm, $z = 10$ mm).

The comparison of the red and green lines clearly indicates the robustness of the proposed approach to changes in excitation positions. In the previous approach, the vibration amplitudes are greatly aggravated compared to the open-loop response. As VCO-MFC does not depend on detailed specifications of *Structure 4* such as excitation positions, the cause of the deteriorations may be the lack of adequate chattering compensation. As shown in Fig. 16, on the other hand, the proposed approach may be applicable to the same structure even when excited at a different position, thanks to the introduction of the effective chattering compensation.

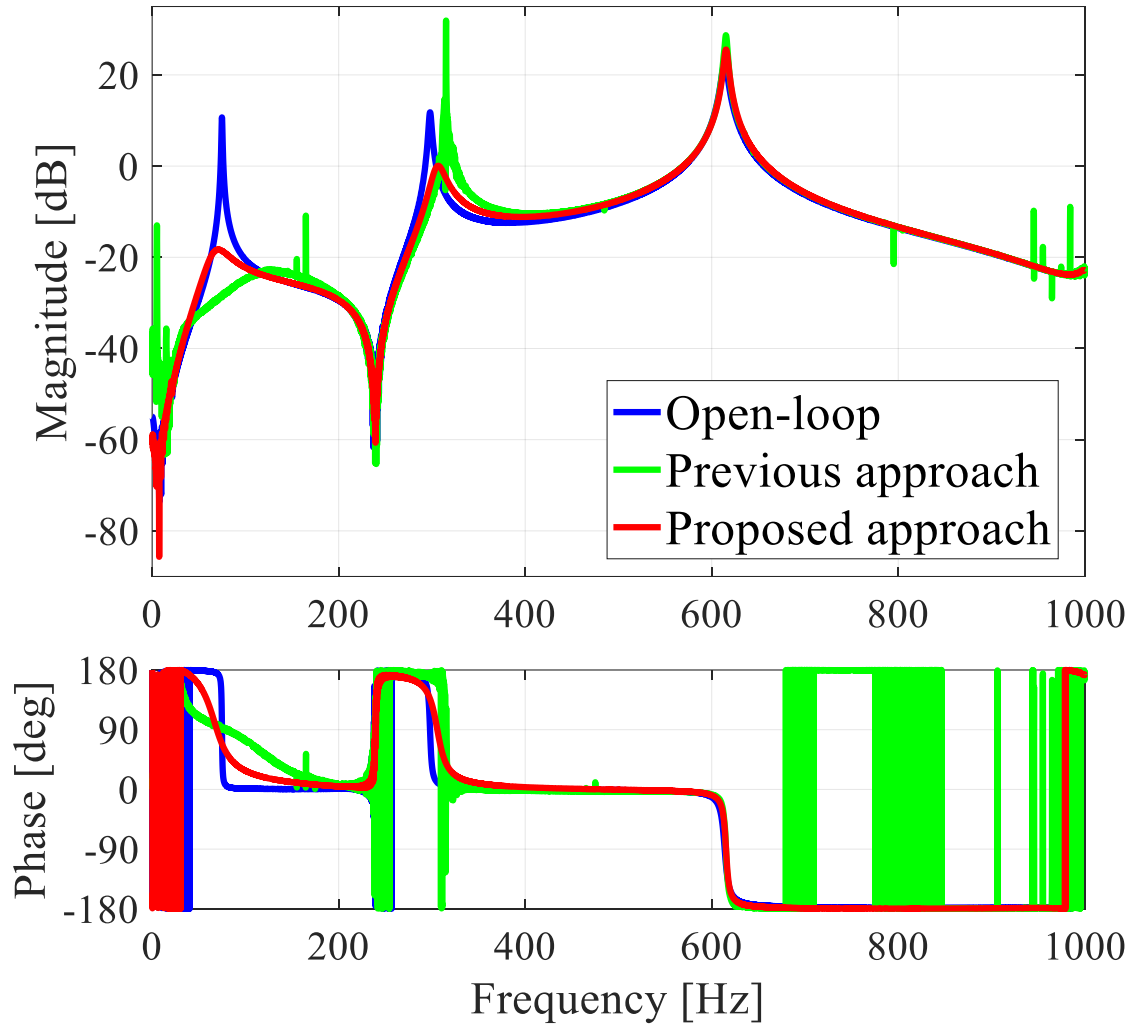


Fig. 16: Frequency responses for *Structure 4* when a 1–1000 Hz linear sweep sinusoidal signal is applied at the changed excitation position ($x = 155$ mm, $y = 250$ mm, $z = 10$ mm).

According to Tables 4, 5, and 6, the damping performance of the proposed approach is almost at the same level as that of the previous approach, except for the chattering issue. Refinement to further optimize the membership functions may allow performance to outperform the current one. The need to carefully adjust the membership functions is one of the limitations of this work and will be further addressed in future study.

According to the experimental mechanism shown in Fig. 5(b) and Fig. 6, the actuator is installed directly at the damping point of the structure, at which the occurred vibrations need to be suppressed, and generates a control force perpendicular to the vibrating surface. The principle of this control system is similar to that of active vibration control in serial-link industrial robots used in precision robotic machining process (robotic milling) [10] and active vibration control via inertial-mass actuators for civil structures [11]. In the application example detailed in [10], the low-stiffness-induced vibrations and

chatter in robotic milling process can be suppressed by directly installing an inertial voice coil actuator, which has the same principle as that shown in Fig. 1, near the source of excitation (e.g., spindle in the end-effector) on the robot to generate control forces. For civil structures, the proposed control architecture has the potential to actively damp vibrations due to human-induced excitations in floor systems [11] with open-plan layouts or lightweight footbridges by placing an inertial-mass actuator on the vibrating surface of the floor. Consequently, the control system proposed in this study is expected to be employed in the specific application scenarios of the real industrial world. The proposed model-free strategy, which does not rely on plant modeling, should be particularly useful for the above applications where modeling is difficult and the working condition always changes depending on the robot pose and its underlying stiffness.

6. Conclusion

In this work, a new approach to construct a model-free FSMC was presented for active damping of mechanical system vibrations. The proposed VCO-MFFSMC does not require any mathematical modeling procedures or parameters of the real controlled object, which is made possible simply by the introduction of VCO for controller design. This model-free scheme was directly connected to SMC, involving convergence analysis via a Lyapunov function approach. Moreover, to tackle the chattering issue, this work constructed a fuzzy inference-based adaptive compensation system. For the VCO-SMC, fuzzy inference was formulated using a Tsukamoto-type method to obtain three benefits simultaneously: elimination of chattering, convergence of state trajectory, and reduction of online computational effort. This work conducted experimental verifications to test the effectiveness of the proposed approach for cantilever plates with characteristic variations. The comparative results confirmed good damping effects and robustness of the proposed approach as well as superiority compared to existing model-free SMC in terms of chattering suppression.

In the future, the present model-free FSMC will be further enhanced in terms of the robustness with consideration of actuator uncertainty and noise tolerance. More detailed robustness analysis for a controlled object with different materials will also be included in future works of this research.

CRedit authorship contribution statement

Heisei Yonezawa: Conceptualization, Methodology, Software, Validation, Investigation, Writing - Original Draft. **Ansei Yonezawa:** Methodology, Software, Validation, Writing - Original Draft. **Itsuro Kajiwara:** Conceptualization, Writing - Original Draft, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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