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Redshift-space distortion constraints on the neutrino mass and models to alleviate the Hubble tension

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Abstract

We discuss the neutrino mass and Hubble tension solutions and examine their effects on the redshift-space distortion (RSD) observations. An analysis with RSD data indicates smaller amplitude of perturbation. Including RSD data results in a slightly weaker upper limit on the neutrino mass than that derived for data without RSD, which is common in other extended models too. We have evaluated the impacts of RSD observations on some extended models, including the varying electron mass model, a time-dependent dark energy model with two parameter equations of state (EOS), and a model where the number of neutrino species is free. When we estimate the cosmological parameters for data including RSD, we found that the EOS parameter for dark energy is larger than that of the cosmological constant, and the effective number of neutrino species is smaller than the standard value, which infers a smaller present Hubble parameter H_0 . From the viewpoint of cosmological tensions, the varying electron mass model with nonzero neutrino mass option looks promising to relax the Hubble tension and the S_8 tension simultaneously.

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I. INTRODUCTION

The standard cosmology with cold dark matter (CDM) and cosmological constant Λ , called the Λ CDM model, is a great way to explain the large scale properties of our Universe. The matter power spectrum, also known as the galaxy two-point correlation function, is the primary quantity that describes the properties of large scale structures. This can provide us with information about the growth of the density perturbation. The redshift of galaxies is measured by galaxy surveys. The positions of galaxies differ at the redshift-space and real position actually, which is referred to as redshift space distortions (RSD) [1, 2]. This distortion originates from not only the cosmic Hubble expansion but also the peculiar velocity of an individual galaxy.

By measuring the RSD, we may find the growth rate of the matter fluctuation [3–12], as a function of redshift z . To be precise, it is expressed by the value $f(z)\sigma_8(z)$, where $f \equiv d \ln \delta_m / d \ln a$ is the growth rate, $\delta_m(z) \equiv \delta \rho_m(z) / \bar{\rho}_m(z)$ is the density contrast, a is the scale factor of our Universe, σ_8 is the amplitude of mass fluctuations in spheres of radius $8h^{-1}$ Mpc, and h is the dimensionless Hubble parameter of $H_0 = 100h$ km/s/Mpc. The growth rate reported in those observations is lower than that inferred by Planck [13] in the CDM model. Numerous extended models have been proposed to explain the discrepancy. See, for instance, Refs. [14–26] for the recent studies.

Another cosmological tension on the standard Λ CDM model under debate is so-called the Hubble tension, which is discrepancy of the present Hubble parameter H_0 measured by local observations [27–33] and inferred from cosmological observations such as cosmic microwave background radiation (CMB) [13] and baryon acoustic oscillation (BAO) [34–36]. While no satisfactory cosmological model has been proposed yet, some models can alleviate the Hubble tension significantly. For a recent review, see e.g., Refs. [37–42].

In this paper, we examine simple extensions of Λ CDM and models that have been proposed to alleviate the Hubble tension and evaluate their effects and constraints from the RSD data. This paper is organized as follows. After we describe the models to be discussed in this paper in the Sec. II, we explain the method and datasets of our analysis in the Sec. III and present the results in the Sec. IV. We summarize this paper in the Sec. V.

II. MODELS

In this section, we give an explanation of the four models we examine and discuss their effects on the observations. We consider two different options for neutrino mass when considering each model. One is to fix the neutrino mass, while the other is to vary the sum of neutrino masses $\sum m_\nu$.

Neutrino mass

When we fix the neutrino mass, we assume the smallest neutrino mass in the spectrum of normal mass ordering $m_\nu = (0, 0, 0.06)$ eV. On the other hand, when we vary the neutrino mass, we vary the total neutrino mass $\sum m_\nu$ with keeping the normal mass ordering and the neutrino energy density given as $\Omega_\nu h^2 = \sum m_\nu / (94.07 \text{ eV}) \times (3.046/3)^{3/4}$ in the standard ($N_{\text{eff}} = 3.046$) case. As the neutrino mass increases, the growth rate of the matter fluctuations with a wave number larger than the free-streaming wave number is suppressed (See Refs. [43–49] for the recent works of the neutrino mass and large scale structure and Refs. [50, 51] for the review). Since the lower growth rate $f\sigma_8$ are reported than assuming Λ CDM best-fit values, we expect that a larger neutrino mass is preferred from the RSD data

¹.

A. Λ CDM

The standard cosmological model has the six free parameters: the baryon density $\omega_b = \Omega_b h^2$, the CDM density $\omega_c = \Omega_c h^2$, the amplitude of the primordial density perturbation A_s , the spectral index of the primordial density perturbation n_s , the acoustic scale $\theta_* = r_*/D_M$ with r_* the sound horizon at recombination and D_M the angular diameter distance to the last scattering surface, and the optical depth of the reionization τ .

¹ When we focus the RSD data only from SDSS-IV [52] which has reported a larger $f\sigma_8$, a lower neutrino mass seems to be preferred [53]. There are also recent apparent unphysical indication of $\sum m_\nu < 0$ from the DESI BAO [54, 55].

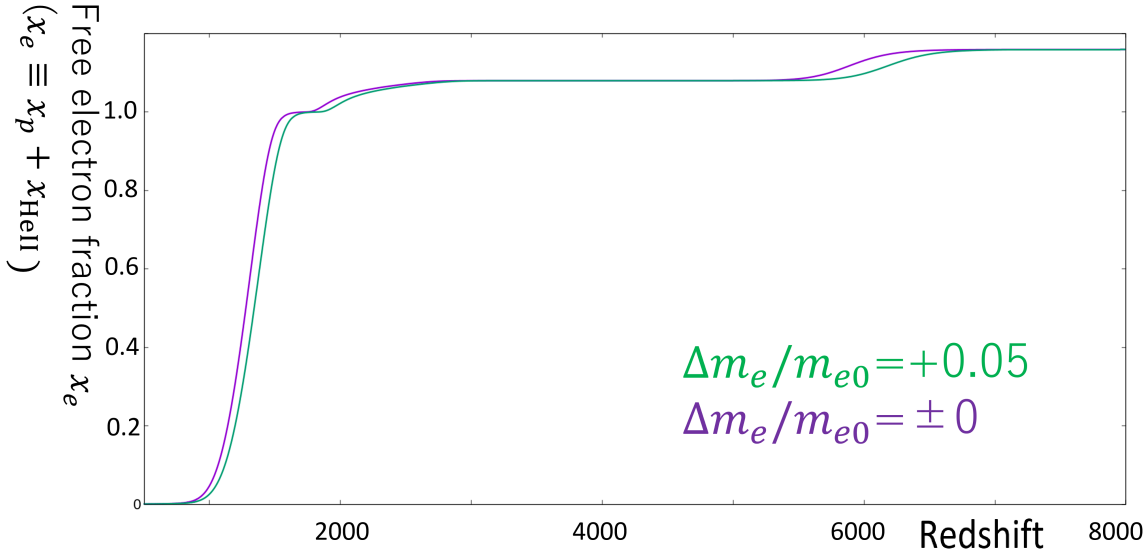


Figure 1: Recombination.

B. Varying electron mass

The varying electron mass model [56] has one additional parameter: m_e/m_{e0} , where m_e is the electron mass in the recombination era and $m_{e0} = 0.511$ MeV is the current electron mass. In this paper, we consider a scenario where $m_e/m_{e0} > 1$ at the early Universe and the value of electron mass changes to the current value after the recombination is complete.

The varying electron mass model is a model that is highly promising for relaxing the Hubble tension [39, 57–59] and has been studied from various aspects [60–64]. As is explained in the Planck(2015) [65], the main contributions of a larger electron mass are the higher energy levels of hydrogen and Lyman alpha photons, both of which are directly proportional to the electron mass² as $E \propto m_e$. Since the energy level of hydrogen is higher with a larger electron mass, a photon with a temperature below standard recombination cannot excite it. Therefore, in the case that the electron mass in the recombination era is larger than today, the recombination occurs earlier than in the standard case as we show in Fig. 1. Then, the earlier recombination leads to a higher Hubble constant H_0 and a smaller Ω_m to reproduce the measured CMB power spectrum as is observed.

² In fact, we also include the minor corrections: Thomson scattering rate, the photoionization cross sections, the recombination coefficient, K factors, Einstein A coefficients, and the two-photon decay rates (see Refs. [63, 65, 66] for the detail).

C. $w_0 w_a$ DE

The dark energy model has two additional parameters: the dark energy equation of state at the present time w and its derivative with respect to the scale factor $w_a = dw/da$. We adopt the so-called CPL parametrization [67, 68] of the equation of state for a time-dependent dark energy as

$$w(a) = w + w_a \left(1 - \frac{a}{a_0}\right) = w + \frac{z}{1+z} w_a, \quad (1)$$

where a_0 is the present value of scale factor a . In this paper, we only consider this parametrization, not other extended models of dark energy. It should be noted that decreasing w leads to an increase in the Hubble constant H_0 [13, 69–71] and an increased $f\sigma_8$, as shown in Fig. 3 of Ref. [72].

D. Extra radiation N_{eff}

Extra radiation model has one additional parameter: N_{eff} which is the effective number of neutrinos defined as

$$\Omega_r = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right) \Omega_\gamma. \quad (2)$$

Here, γ and r denote CMB photons and radiation, respectively. We consider the extra radiation model to be a typical example of models where the energy density of other components is substantial during the early Universe, such as early dark energy [73–84]. We note that a large ΔN_{eff} can be easily compatible with big bang nucleosynthesis if there is large lepton asymmetry [85] or assuming the ΔN_{eff} production after BBN [86].

As in the default setting of CAMB, in our calculation, we assume that the energy density of neutrinos is expressed by

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{94.07 \text{ eV}} \left(\frac{3.046}{3}\right)^{3/4} \quad (N_{\text{eff}} > 3.046), \quad (3)$$

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{94.07 \text{ eV}} \left(\frac{N_{\text{eff}}}{3}\right)^{3/4} \quad (\text{else}). \quad (4)$$

Such positive ($\Delta N_{\text{eff}} > 0$) is a case if additional massless particles besides neutrinos are produced, whereas negative ($\Delta N_{\text{eff}} < 0$) is realized by an additional entropy production after neutrino decoupling.

III. DATASETS AND METHODOLOGY

We conduct a Markov-chain Monte Carlo (MCMC) analysis of four models using the public MCMC code `CosmoMC-planck2018` [87] to examine the effects of each model on a variety of cosmological observations with and without varying neutrino mass. We require the convergence $R - 1 < 0.03$ and analyze the models by referring to the following cosmological observation. We always used the CMB, BAO and light curve of SNeIa data, and we call this dataset $\mathcal{D}$.

- CMB from Planck [13]. We use the temperature and polarization likelihoods for high l `plik` ($l = 30$ to 2508 in TT and $l = 30$ to 1997 in EE and TE) and low l `Commander` and lowE `SimAll` ($l = 2$ to 29). We also include CMB lensing [88].
- BAO distance data from 6dF [34], MGS [35], DR12 [36], and DESI BAO [54] (specific values are listed in Table I).
- Light curve of SNeIa from *Pantheon* [89].
- Priors on the values of $f\sigma_8(z)$ from the several observations [3–12] (specific values are listed in Table II). The covariance matrix of the RSD data is diagonal because the correlation coefficients are currently not available. Hereafter, we call this “RSD”.
- A prior on the Hubble constant ($H_0 = 73.30 \pm 1.04$ km/sec/Mpc at 68% CL) from the SH0ES Collaboration [27]. Hereafter, we call this “R21”.

IV. RESULTS

In the Table III, we summarize the constraints of the cosmological parameters and the Gaussian tension which is defined as,

$$T_{H_0} = \frac{H_0 \mathcal{D} + \text{RSD} - 73.30 \text{ km/s/Mpc}}{\sqrt{\sigma_{\mathcal{D} + \text{RSD}}^2 + (1.04 \text{ km/s/Mpc})^2}}, \quad (5)$$

for the Hubble tension with the direct measurement of the Hubble constant [27] and

$$T_{S_8} = \frac{S_8 \mathcal{D} + \text{RSD} - 0.776}{\sqrt{\sigma_{\mathcal{D} + \text{RSD}}^2 + \frac{0.017^2 + 0.017^2}{2}}}, \quad (6)$$

z_{eff}		z_{eff}	
6DF [34]		DESI [54]	
0.106	$r_s/D_V=0.336\pm0.015$	0.30	$D_V/r_d=7.93\pm0.15$
		0.51	$D_M/r_d=13.62\pm0.25$
MGS [35]		0.51	$D_H/r_d=20.98\pm0.61$
0.105	$D_V/r_s=4.466\pm0.168$	0.71	$D_M/r_d=16.85\pm0.32$
		0.71	$D_H/r_d=20.08\pm0.60$
DR12 [36]		0.93	$D_M/r_d=21.71\pm0.28$
0.38	$D_M/r_s=1512.39\pm24.99$	0.93	$D_H/r_d=17.88\pm0.35$
0.38	$H(z)r_s=81.2087\pm2.3683$	1.32	$D_M/r_d=27.79\pm0.69$
0.51	$D_M/r_s=1975.22\pm30.10$	1.32	$D_H/r_d=13.82\pm0.42$
0.51	$H(z)r_s=90.9029\pm2.3288$	1.49	$D_V/r_d=26.07\pm0.67$
0.61	$D_M/r_s=2306.68\pm37.08$	2.33	$D_M/r_d=39.71\pm0.94$
0.61	$H(z)r_s=98.9647\pm 2.5019$	2.33	$D_H/r_d=8.52\pm0.17$

Table I: The distance and expansion rete data from 6DF, MGS, and DR12 (left). The distant data from DESI (right).

for the S_8 tension with DES [90]. We define $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$.

In the Table IV, we summarize the best-fit values of the cosmological parameters and their corresponding minimized chi-squared values. To penalize increasing the number of parameters and facilitate fair comparison, we also calculate the Akaike information criterium (AIC) of model X relative to that of Λ CDM model as follows:

$$\Delta AIC = \chi_{\text{min},X}^2 - \chi_{\text{min},\Lambda\text{CDM}}^2 + 2(N_X - N_{\Lambda\text{CDM}}), \quad (7)$$

where $N_X - N_{\Lambda\text{CDM}}$ is the number difference in the free parameters between the model X and Λ CDM model.

Survey	z	$f(z)\sigma_8(z)$	References
ALFALFA	0.013	0.46 ± 0.06	[12]
6dFGS+SDSS	0.035	0.338 ± 0.027	[10]
GAMA	0.18	0.29 ± 0.10	[6]
	0.38	0.44 ± 0.06	[5]
WiggleZ	0.22	0.42 ± 0.07	[4]
	0.41	0.45 ± 0.04	
	0.60	0.43 ± 0.04	
	0.78	0.38 ± 0.04	
DR12 BOSS	0.32	0.427 ± 0.056	[8]
	0.57	0.426 ± 0.029	
VIPERS	0.60	0.49 ± 0.12	[9]
	0.86	0.46 ± 0.09	
VVDS	0.77	0.49 ± 0.18	[3]
FastSound	1.36	0.482 ± 0.116	[7]
eBOSS Quasar	1.48	0.462 ± 0.045	[11]

Table II: Specific values of $f(z)\sigma_8(z)$ and their errors for each measurements.

A. Λ CDM

The 68% constraints and best-fit values of the cosmological parameters on Λ CDM model and the Λ CDM + m_ν model are summarized in Table III and Table IV. The one-dimensional posterior distributions and two-dimensional allowed regions are shown in Fig. 2.

The inclusion of RSD data results in the preference of a higher Hubble constant and lower S_8 , which alleviate both the Hubble tension and the S_8 tension, for the simple Λ CDM (fixed neutrino mass) model as well as to allow a larger neutrino mass for the Λ CDM + m_ν

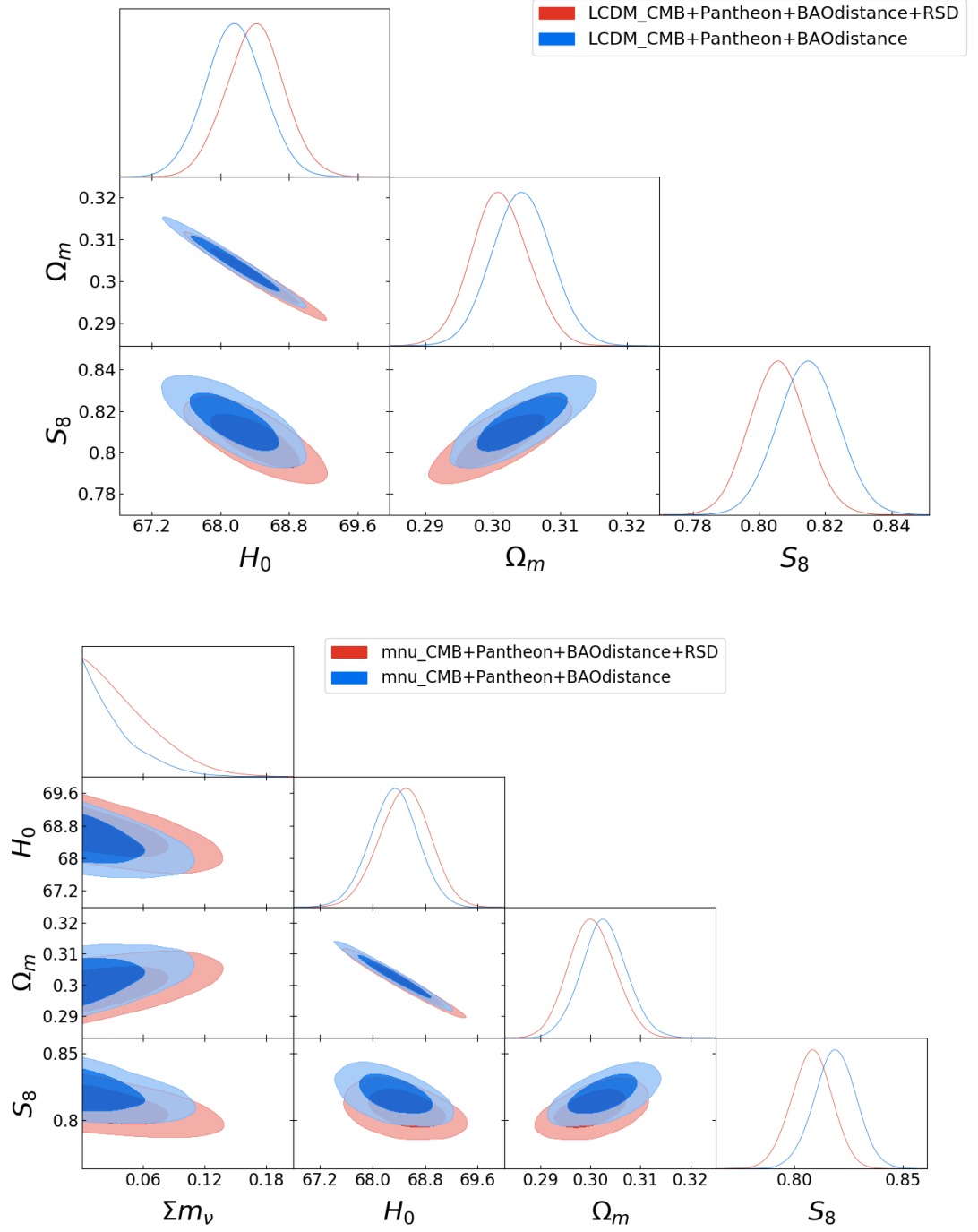


Figure 2: Posterior distributions of the several parameters on Λ CDM model and Λ CDM + m_ν model for the data $\mathcal{D}$ with/without RSD.

model, as we discussed in Sec. II. We obtain the 95% upper limits of neutrino mass as,

$$\sum m_\nu < 0.0880 \text{ eV} \quad (\mathcal{D}), \quad (8)$$

$$\sum m_\nu < 0.111 \text{ eV} \quad (\mathcal{D} + \text{RSD}). \quad (9)$$

Both inequalities (8) and (9) are consistent with the fact that the sum of neutrino mass is larger than 0.06 eV from observations of neutrino oscillation.

B. Varying m_e

The 68% constraints and best-fit values of the cosmological parameters on the varying m_e model and the varying $m_e + m_\nu$ model are summarized in Table III and Table IV. The one-dimensional posterior distributions and two-dimensional allowed regions are shown in Fig. 3.

For the varying m_e (fixed neutrino mass) model, when we include RSD data, we find that a smaller electron mass and a slightly lower Hubble constant are preferred as,

$$\begin{aligned} m_e/m_{e0} &= 1.0092 \pm 0.0055, \\ H_0 &= 69.44 \pm 0.84 \text{ km/s/Mpc} \quad (\mathcal{D}), \end{aligned} \quad (10)$$

and

$$\begin{aligned} m_e/m_{e0} &= 1.0058 \pm 0.0053, \\ H_0 &= 69.23 \pm 0.83 \text{ km/s/Mpc} \quad (\mathcal{D} + \text{RSD}). \end{aligned} \quad (11)$$

It is understood, as follows, that a larger electron mass m_e/m_{e0} also leads to too large σ_8 and too large growth rate $f\sigma_8(z)$ to fit with the RSD data. This can also be seen by the Table IV and Fig. 6. In the Table IV, though the varying m_e model greatly improves the total chi-squared values, the chi-squared value of RSD becomes worse from $\chi^2_{\text{RSD}} = 22.572$ in Λ CDM model to $\chi^2_{\text{RSD}} = 28.609$ in varying m_e model. In the Fig. 6, we show a growth rates $f\sigma_8(z)$ as a function of redshift, and we draw the blue curve using the best-fit values of the varying m_e model and the orange curve using the best-fit values of Λ CDM model.

On the other hand, a larger neutrino mass is allowed in the varying $m_e + m_\nu$ model when we include the RSD data. It is fascinating to observe that a larger neutrino mass leads to a

larger electron mass, which in turn leads to a higher Hubble constant. Thus, we obtain the 68% constraints of m_e/m_{e0} and H_0 and the 95% upper limits of neutrino mass as,

$$\begin{aligned} m_e/m_{e0} &= 1.0094^{+0.0059}_{-0.0077}, \\ H_0 &= 69.45^{+0.85}_{-0.95} \text{ km/s/Mpc}, \\ \sum m_\nu &< 0.193 \text{ eV} \quad (\mathcal{D}), \end{aligned} \tag{12}$$

$$\begin{aligned} m_e/m_{e0} &= 1.0114^{+0.0069}_{-0.011}, \\ H_0 &= 69.66^{+0.93}_{-1.1} \text{ km/s/Mpc}, \\ \sum m_\nu &< 0.330 \text{ eV} \quad (\mathcal{D} + \text{RSD}). \end{aligned} \tag{13}$$

It is worth noting that the varying $m_e + m_\nu$ model is the best solution to both the Hubble and S_8 tension as far as we see in Table III.

C. $w_0 w_a$ DE

The 68% constraints and best-fit values of the cosmological parameters in the $w_0 w_a$ DE model and $w_0 w_a$ DE + m_ν model are summarized in Table III and Table IV. The one-dimensional posterior distributions and two-dimensional allowed regions are illustrated in Fig. 4.

For the varying $w_0 w_a$ DE (fixed neutrino mass) model, the recent DESI paper [54] discovered that the dynamical ($w > -1$) dark energy is preferred at a significance level of approximately 2.6σ using CMB(Planck and Atacama Cosmology Telescope data), Pantheon+ [91] and DESI BAO data. We also find the preference of the dynamical dark energy as,

$$\begin{aligned} w &= -0.919 \pm 0.069, \\ w_a &= -0.42 \pm 0.25, \\ H_0 &= 68.44 \pm 0.75 \text{ km/s/Mpc} \quad (\mathcal{D}), \end{aligned} \tag{14}$$

$$\begin{aligned} w &= -0.900 \pm 0.069, \\ w_a &= -0.38 \pm 0.24, \\ H_0 &= 68.04 \pm 0.72 \text{ km/s/Mpc} \quad (\mathcal{D} + \text{RSD}). \end{aligned} \tag{15}$$

Including the RSD data results in a preference for larger w and w_a because a larger w results in a smaller $f\sigma_8$ and is consistent with the observed RSD. By having a larger w , the Hubble constant decreases, which takes the Hubble tension more seriously. In the Table III, we show that the center values of the Hubble constant get lower than the Λ CDM case, although the tensions T_{H_0} are slightly improved due to larger errors.

For the varying $w_0 w_a$ DE + m_ν model, it is worth mentioning that a larger neutrino mass is allowed than Λ CDM case (Eqs. (8) and (9)) and its 95% upper limits are given by

$$\sum m_\nu < 0.159 \text{ eV} \quad (\mathcal{D}), \quad (16)$$

$$\sum m_\nu < 0.195 \text{ eV} \quad (\mathcal{D} + \text{RSD}). \quad (17)$$

D. Extra radiation N_{eff}

The 68% constraints and best-fit values of the cosmological parameters on extra radiation N_{eff} model and extra radiation $N_{\text{eff}} + m_\nu$ model are summarized in Table III and Table IV. The one dimensional posterior distributions and two-dimensional allowed regions are illustrated in Fig. 5.

For both the N_{eff} model and $N_{\text{eff}} + m_\nu$ model, when we include RSD data, we find that a smaller N_{eff} , a lower Hubble constant H_0 , and a lower S_8 are preferred. As in Table IV, in the N_{eff} model and $N_{\text{eff}} + m_\nu$ model, the best-fit values of the Hubble constant are around 70 km/s/Mpc, and they do not get the value of χ^2_{RSD} so worse as the varying m_e model does.

V. CONCLUSIONS

In this paper, we examine several cosmological models and how they affect observations, with particular emphasis on RSD.

The analysis with RSD data reveals that S_8 and $f\sigma_8$ are smaller, which can be seen in the results for Λ CDM in Fig. 2. Adding RSD allows for additional parameters and effects to reduce σ_8 , including the neutrinos mass. In fact, when we include the RSD data into analysis, we found a slightly weaker 95% upper limit $\sum m_\nu < 0.111 \text{ eV}(\mathcal{D} + \text{RSD})$ than the 95% upper limit $\sum m_\nu < 0.0880 \text{ eV}(\mathcal{D})$, which is very close to the bound $\sum m_\nu \geq 0.06 \text{ eV}$

from the neutrino oscillation. The inclusion of RSD results in a weaker neutrino mass bound in other extended models as shown in Figs. 3, 4 and 5.

Motivated by the Hubble tension, we have evaluated the impacts of RSD observations on the varying m_e model, w_0w_a model and ΔN_{eff} model. For data $\mathcal{D} + \text{RSD}$, only varying m_e model is compatible with a higher H_0 , while the others fit with a smaller H_0 than that in ΛCDM model, because a larger EOS parameters of dark energy or smaller N_{eff} than 3 are preferred in those models. Hence, the Hubble tension is not improved.

The best-fit points for data $\mathcal{D} + \text{RSD} + \text{R21}$ are $H_0 \simeq 70$ km/s/Mpc for the ΔN_{eff} model and $H_0 \simeq 71$ km/s/Mpc for the varying m_e model. As far as $\Delta\chi_{\text{RSD}}^2$ is concerned, χ_{RSD}^2 is worsened by about 1 in the ΔN_{eff} model and by approximately 6 in the varying m_e model, respectively. Since the variation of χ_{R21}^2 is more than those of RSD and dominate, the total are opposite to the response for χ_{RSD}^2 . This is nothing but none of those relax the Hubble tension and the amplitude of fluctuation simultaneously. This can be understood as follows. To alleviate the Hubble tension by shortening the sound horizon, the value of σ_8 increases while the matter density Ω_m decreases. In Fig. 6, we plot the matter growth rate $f\sigma_8$ for the best-fit of the ΛCDM model (orange curve) and the best-fit of the varying electron mass model (blue curve). As shown in the figure, in the low-redshift region ($0 < z < 1$), the blue curve deviates more significantly from the observational error bars compared to the orange curve. It is the reason why the value of χ_{RSD}^2 gets worse in the N_{eff} model and the varying m_e model.

In summary, Fig. 7 shows the model comparison regarding constraints on the neutrino mass and Hubble constant, newly obtained in this paper, in the analysis with the $\mathcal{D} + \text{RSD}$ data. At a glance, it is apparent that the varying $m_e + m_\nu$ model permits both the largest neutrino mass and highest Hubble constant among the four models.

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Parameter	LCDM	LCDM+ $\sum m_\nu$	me	me+ $\sum m_\nu$
$\Omega_b h^2$	0.02255 ± 0.00013	0.02253 ± 0.00013	0.02261 ± 0.00014	$0.02271^{+0.00017}_{-0.00022}$
$\Omega_c h^2$	0.11770 ± 0.00074	0.11784 ± 0.00076	0.1194 ± 0.0017	$0.1206^{+0.0020}_{-0.0026}$
Σm_ν		< 0.111		< 0.330
m_e/m_{e0}			1.0058 ± 0.0053	$1.0114^{+0.0069}_{-0.011}$
H_0	68.41 ± 0.34	68.49 ± 0.38	69.23 ± 0.83	$69.66^{+0.93}_{-1.1}$
Ω_m	0.3011 ± 0.0043	0.3003 ± 0.0047	0.2978 ± 0.0053	0.2982 ± 0.0054
S_8	0.8058 ± 0.0085	0.8079 ± 0.0094	0.8087 ± 0.0090	$0.804^{+0.013}_{-0.010}$
$r_{\text{drag}} h$	100.90 ± 0.57	101.01 ± 0.62	101.38 ± 0.73	101.36 ± 0.73
$f\sigma_8(0.38)$	0.4672 ± 0.0040	$0.4685^{+0.0048}_{-0.0043}$	0.4700 ± 0.0048	$0.4681^{+0.0064}_{-0.0053}$
$f\sigma_8(0.61)$	0.4630 ± 0.0035	$0.4643^{+0.0046}_{-0.0040}$	0.4664 ± 0.0048	$0.4645^{+0.0064}_{-0.0054}$
T_{H_0}	4.46σ	4.34σ	3.05σ	2.50σ
T_{S_8}	1.56σ	1.64σ	1.70σ	1.36σ
Parameter	$w_0 w_a$ CDM	$w_0 w_a$ CDM+ $\sum m_\nu$	N_{eff}	$N_{\text{eff}} + \sum m_\nu$
$\Omega_b h^2$	0.02249 ± 0.00014	0.02249 ± 0.00013	0.02249 ± 0.00017	0.02246 ± 0.00018
$\Omega_c h^2$	0.11841 ± 0.00094	0.11835 ± 0.00094	0.1165 ± 0.0027	0.1160 ± 0.0027
Σm_ν		< 0.195		< 0.103
w_0, N_{eff}	-0.900 ± 0.069	-0.894 ± 0.067	2.97 ± 0.16	2.93 ± 0.17
w_a	-0.38 ± 0.24	$-0.41^{+0.28}_{-0.24}$		
H_0	68.04 ± 0.72	68.00 ± 0.74	67.9 ± 1.0	67.9 ± 1.0
Ω_m	0.3058 ± 0.0069	0.3066 ± 0.0071	0.3025 ± 0.0050	0.3017 ± 0.0050
S_8	0.8135 ± 0.0099	0.812 ± 0.011	0.8037 ± 0.0094	0.8057 ± 0.0098
$r_{\text{drag}} h$	100.3 ± 1.1	100.2 ± 1.1	100.75 ± 0.63	100.88 ± 0.64
$f\sigma_8(0.38)$	0.4660 ± 0.0065	0.4655 ± 0.0068	0.4656 ± 0.0050	0.4667 ± 0.0052
$f\sigma_8(0.61)$	0.4636 ± 0.0069	0.4632 ± 0.0072	0.4611 ± 0.0050	0.4623 ± 0.0052
T_{H_0}	4.16σ	4.15σ	3.74σ	3.74σ
T_{S_8}	1.91σ	1.78σ	1.43σ	1.51σ

Table III: 95% upper limit of neutrino mass and 68% C.L of the other parameters. We use CMB, Pantheon, BAO distance and RSD data.

Parameter	LCDM	Λ CDM+ $\sum m_\nu$	$w_0 w_a$ DE	$w_0 w_a$ DE+ $\sum m_\nu$	N_{eff}	$N_{\text{eff}}+\sum m_\nu$	m_e	$m_e+\sum m_\nu$
Σm_ν	-	0.0519	-	0.0684	-	0.0434	-	0.0711
$w_0, N_{\text{eff}}, m_e/m_{e0}$	-	-	-1.0041	-1.0109	3.217	3.254	1.018	1.021
w_a	-	-	-0.0113	-0.0115	-	-	-	-
H_0	68.90	68.93	69.03	69.09	69.73	70.08	71.28	71.62
Ω_m	0.2951	0.2949	0.2943	0.2945	0.2945	0.2927	0.2871	0.2865
S_8	0.7975	0.8003	0.7987	0.7994	0.8031	0.8068	0.8165	0.8150
χ^2_{CMB}	2780.696	2780.445	2779.891	2778.858	2780.831	2780.430	2775.648	2776.177
$\chi^2_{\text{Panthoeon}}$	1034.765	1034.767	1034.822	1034.892	1034.775	1034.816	1035.033	1035.062
χ^2_{RSD}	22.572	23.222	23.117	23.609	23.975	24.970	28.609	28.276
$\chi^2_{\text{BAO distance}}$	19.179	19.189	19.424	19.530	19.222	19.613	23.234	23.950
$\chi^2_{H_0}$	17.903	17.666	16.847	16.387	11.778	9.594	3.759	2.619
χ^2_{prior}	2.304	1.502	1.842	1.895	2.048	1.830	1.791	1.731
χ^2_{total}	3877.42	3876.79	3875.94	3875.17	3872.63	3871.25	3868.07	3867.82
$\chi^2_{\text{total}} - \chi^2_{\text{total } \Lambda\text{CDM}}$		-0.63	-1.48	-2.25	-4.79	-6.17	-9.35	-9.6
ΔAIC		1.37	2.52	3.75	-2.79	-2.17	-7.35	-5.6

data $\mathcal{D}$ +RSD+R21.

Table IV: The best-fit values of the cosmological parameters and their corresponding minimized chi-squared values. We use the

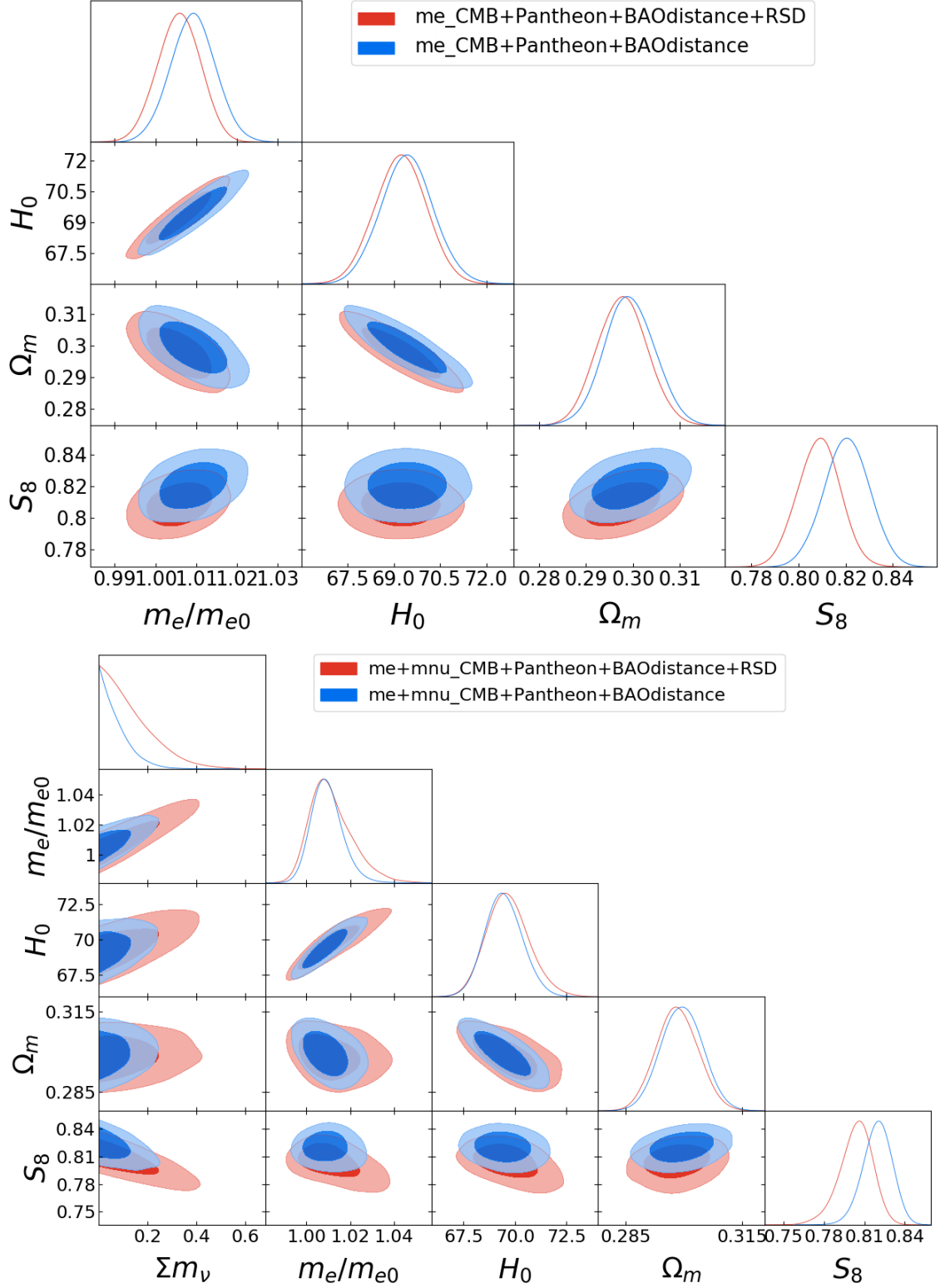


Figure 3: Posterior distributions of the several parameters on varying m_e model and varying $m_e + m_\nu$ model for the data $\mathcal{D}$ with/without RSD.

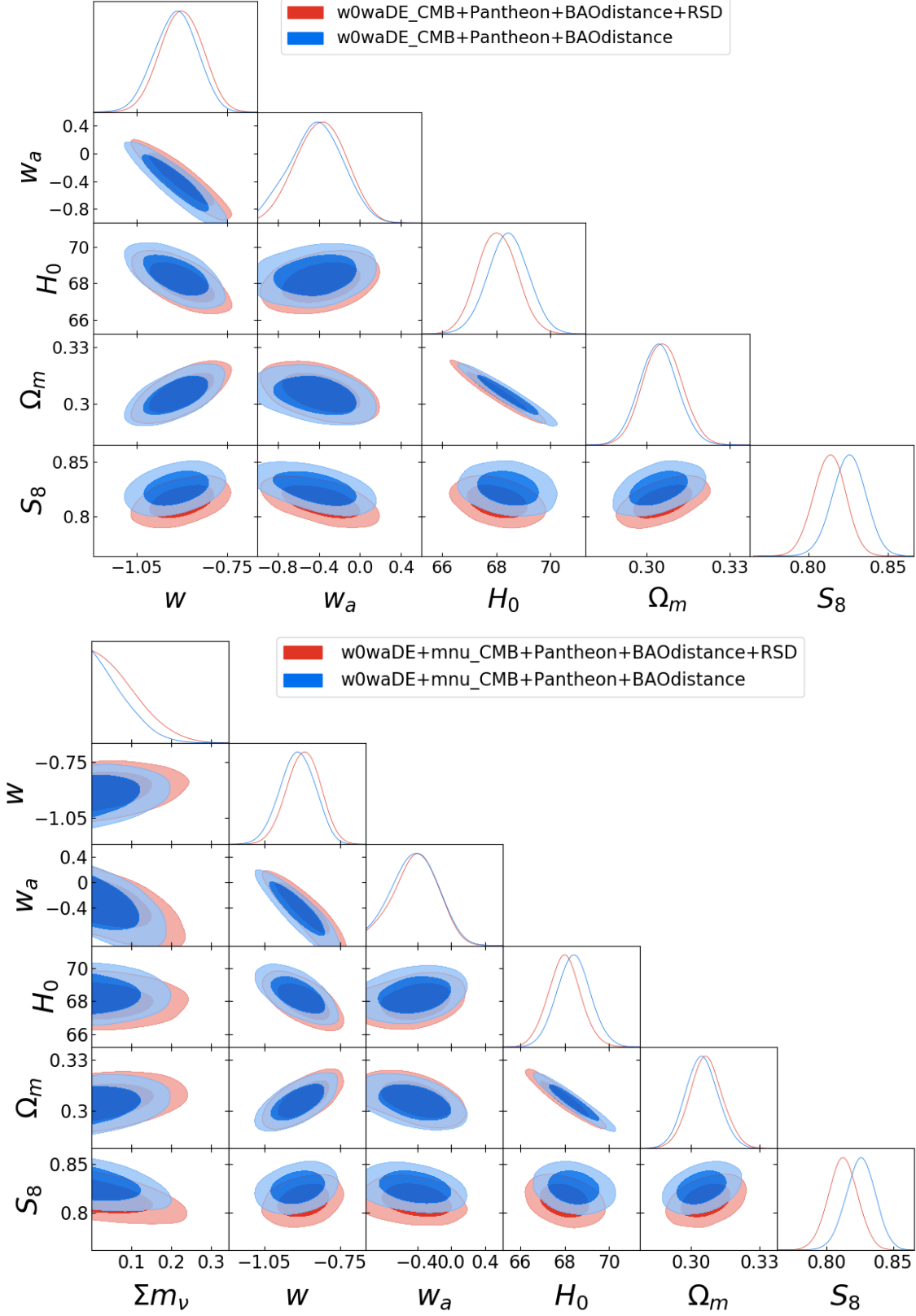


Figure 4: Posterior distributions of the several parameters on $w_0w_a\text{DE}$ model and $w_0w_a\text{DE}+m_\nu$ model for the data $\mathcal{D}$ with/without RSD.

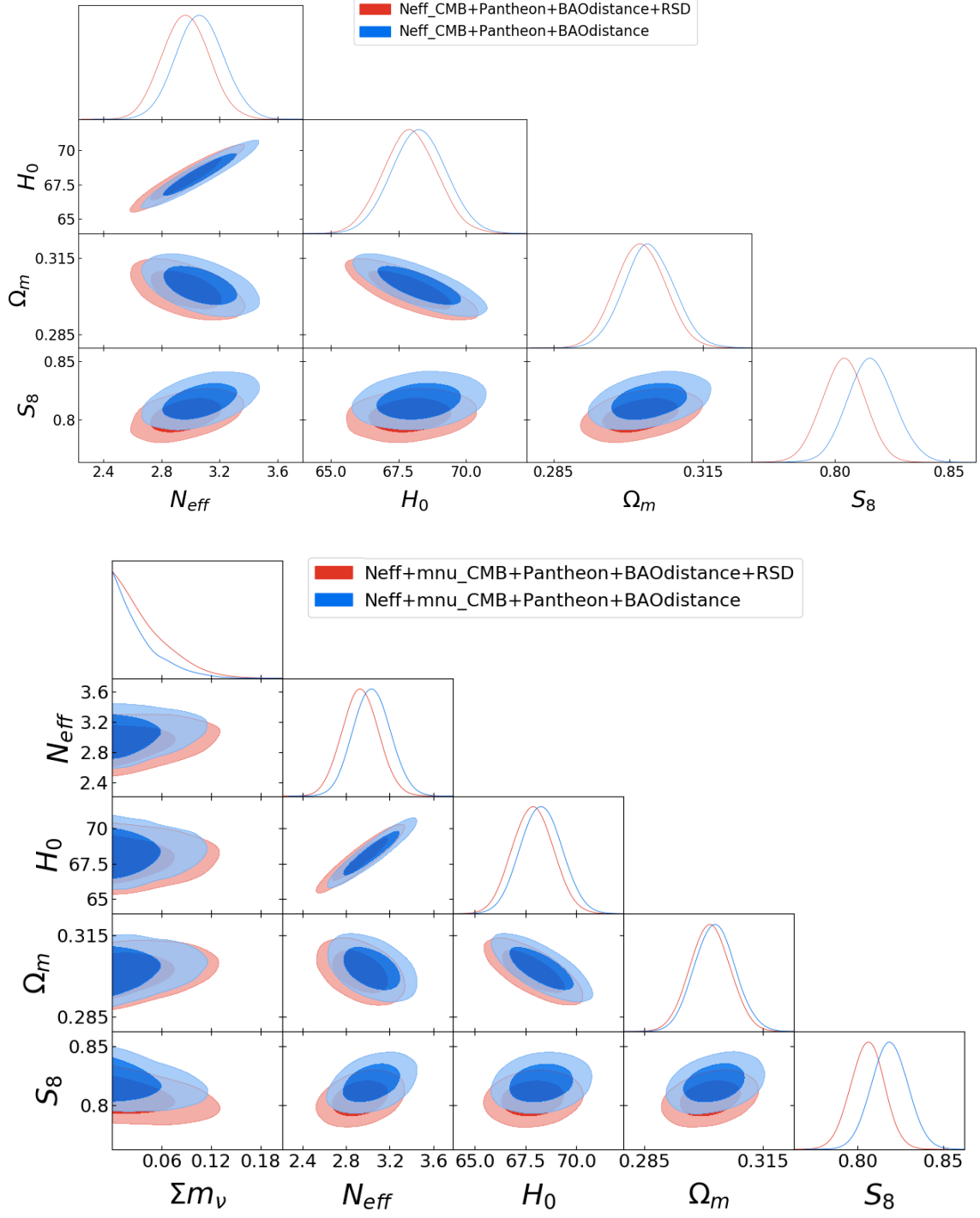


Figure 5: Posterior distributions of the several parameters on N_{eff} model and $N_{\text{eff}} + m_\nu$ model for the data $\mathcal{D}$ with/without RSD.

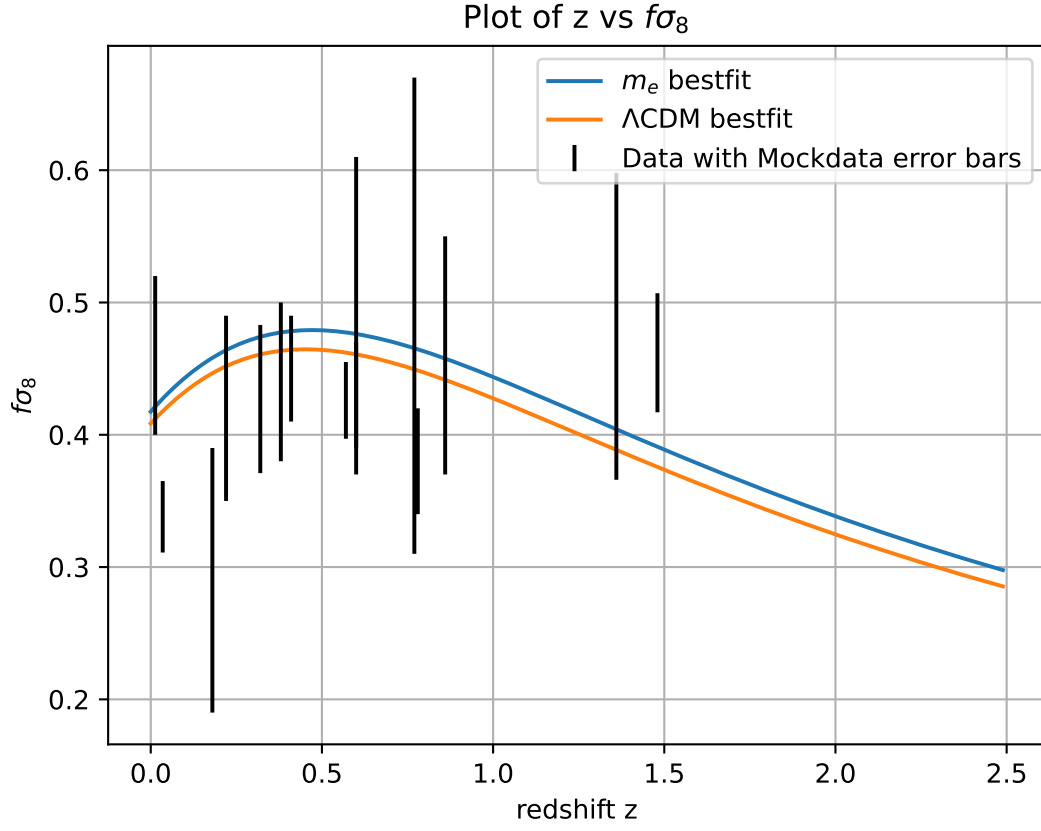


Figure 6: The plot of $f\sigma_8(z)$ for the best-fit value of Λ CDM model $[(\Omega_m, \sigma_8) = (0.2951, 0.8041)]$ and varying electron mass model $[(\Omega_m, \sigma_8) = (0.2871, 0.8347)]$.

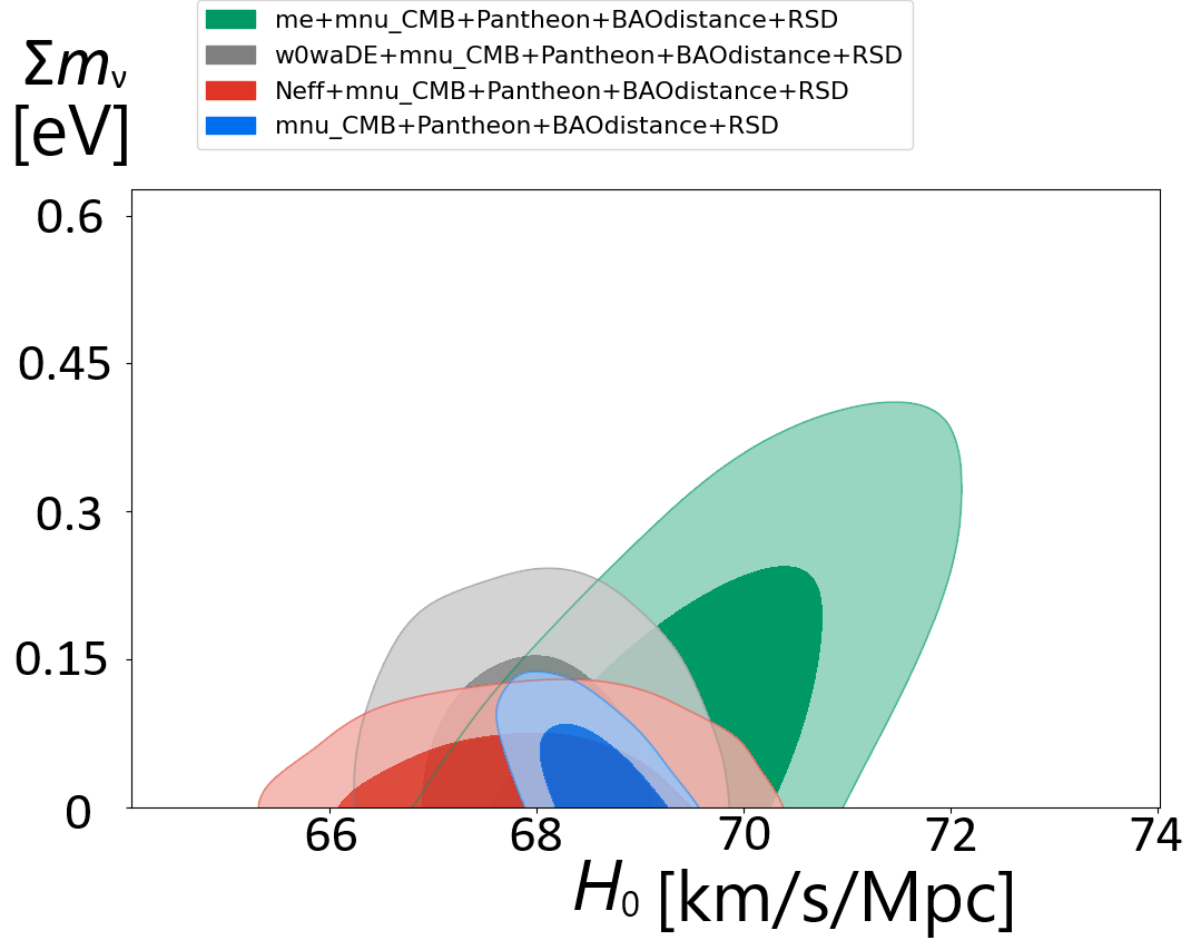


Figure 7: The summary plot of the 2D constraints on Σm_ν and H_0 for the data $\mathcal{D}$ with RSD.