Analytical derivation of charge relaxation time distribution in a transistor from current noise spectrum using inverse integral transformation method

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An analytical technique for extracting relaxation time distribution of dynamic charge events from the current noise spectrum of a transistor is developed by applying an inverse integral transformation to the McWhorter model. In our method, the continuous distribution function of relaxation time $G(\tau)$ can be analytically derived from the noise spectra $S(\omega)$ without spectrum deconvolution. The feasibility of our method is demonstrated by characterizing the charge dynamics of Tetraphenylporphyrin (TPP) molecules dispersed on the surface of a GaAs-based nanowire field-effect transistor (FET). Our analysis successfully verified the time constant of the molecule-related dynamic charge events and effects of photo excitation.

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Current noise in transistors greatly affects the performance of electron circuits and systems. Accordingly, measurement and analysis of noise are indispensable for evaluation and improvement of electronic device capability. Current noise in low frequency region is always dominated by 1/*f* noise whose spectrum occasionally shows bumps or shoulders [1-6]. This shoulder is usually attributed to charging and discharging of the electron trap, which often correlates to the reliability of the device [7, 8]. In general, the shoulder of the noise spectrum arises from discrete charge events showing Lorentzian noise spectrum. Its corner frequency provides the relaxation time of the event, which in turn gives information of the energy level of the trap [9]. Moreover, recently, a novel technique to detect molecular charge dynamics has been developed in terms of noise in electronic devices such as field-effect transistors (FET), semiconductor nanowires [3-5], and carbon nanotube channels [6]. When the molecules are dispersed on the channel surface, Lorentzian noise which is attributed to the random charging and discharging of the discrete traps and having a specific corner frequency is imposed. Conventionally, to extract relaxation time distribution of dynamic charge events, curve fitting of the noise spectrum is performed assuming Lorentz spectral components on the basis of the McWhorter model [10]. However, aside from the fact that a finite number of components should be assumed, it is challenging for this technique to derive continuous relaxation time distribution. In addition, the fitting process often includes arbitrariness. Therefore, in this letter, we develop a noise spectrum analysis technique for deriving the relaxation time distribution from the noise spectrum without any complicated assumption of an analytical function for distribution and fitting. We demonstrate the feasibility of our technique in the analysis on the charge dynamics on molecules dispersed on a GaAs-based nanowire FET working as a charge sensor.

Figure 1 illustrates our experimental setup and a typical example of a measured drain current noise spectra in the GaAs-based nanowire FET. The device shown in **Fig. 1(a)** was

fabricated on an AlGaAs/GaAs heterostructure with a two-dimensional electron gas (2DEG). The length of the PtPd Schottky gate was 400 nm. Tetraphenylporphyrin (TPP) was dispersed on the device surface as target molecules. TPP is known as an electroluminescent layer for organic electroluminescent diodes [11, 12] and a photo-excited donor in the dye-sensitized solar cells [13, 14]. The main light absorption band of TPP, the so called Soret band, is around 419 nm [15]. TPP was found to serve as a photo-excited donor in GaAs-based nanowire [5, 16], when the irradiation wavelength overlaps the Soret band. For exciting the charge in TPP, a light-emitting diode (LED) with the wavelength of 405 nm, which overlaps the tail of the Soret band, was used. Details of the experiment are given in Ref. [5]. **Figure 1(b)** shows the measured drain current noise power spectra $S_{IDS}(f)$. Under dark condition, $1/f$ noise was dominant, similar to conventional FETs. However, irradiating light on the FET with TPP, a shoulder appeared in the spectrum and it became more prominent with increasing irradiation light power. On the other hand, the noise spectrum for the device without TPP was hardly affected by light irradiation (not shown here) [5]. These results indicate that charge transfer stochastically occurred between molecules and the device. In addition, with increasing irradiation power, the position of the shoulder shifted towards the higher frequency direction.

To describe the relaxation time distribution in detail, it is crucial to define the distribution directly from the experimentally obtained power spectrum. We then deduce the relaxation time distribution in the context of our inverse integral transformation method. Here, we assume that McWhorter model is applicable [10]. Namely the power spectrum of $S(\omega)$ as a function of angular frequency $\omega = 2\pi f$, can be obtained by the combination of Lorentzian functions having different relaxation time constants τ:

$$S(\omega) = \int_{\tau_1}^{\tau_2} g(\tau) \frac{\tau}{1+\omega^2\tau^2} d\tau , \qquad (1)$$

where $g(\tau)$ is the statistical weight, and τ_1 and τ_2 are the limits of the integration. Integrating

Eq. (1) with power-law distribution $g(\tau) \propto 1/\tau^{(2-\alpha)}$ gives the power spectrum $S(\omega) \propto 1/\omega^{\alpha}$ [17, 18]. To cover the wide time range of the events, we define the distribution in a logarithmic time scale instead of a linear time scale. The interval of integration $[\tau_1, \tau_2]$ in Eq. (1) is also replaced by $[-\infty, \infty]$. The spectrum based on McWhorter model is then transformed to the equation,

$$S(\omega) = \int_{-\infty}^{\infty} G(\tau) \frac{\tau}{1+\omega^2\tau^2} d\ln\tau . \quad (2)$$

$G(\tau)$ relates to the statistical weight $g(\tau)$ through the equation $G(\tau) = \tau g(\tau)$. $G(\tau)$ can be obtained by calculating the inverse integral transform of $S(\omega)$ in Eq. (2). In a previous recent work, we have deduced an approximate solution for extracting $G(\tau)$ from $S(\omega)$ [19]. Aiming for a more precise calculation, in this present work, we propose a generalized and exact analytical solution of $G(\tau)$ (see Appendix):

$$G(\tau) = \frac{1}{\pi\tau}\{\mathrm{Im}[S(-i/\tau)] - \mathrm{Im}[S(i/\tau)]\}. \quad (3)$$

Eq. (3) represents the inversion form of the spectrum described by McWhorter model. If an appropriate analytical form of $S(\omega)$ is available, $G(\tau)$ can be directly obtained by calculating Eq. (3). Owing to the linearity of the transform, a series expansion technique can be used for the analysis. When the spectrum is expressed by linear combination of analytical functions, $S_j(\omega)$ (j = 1, 2, ..., n), the relaxation time distribution is given by the series of $G_j(\tau)$. Each of these $G_j(\tau)$ is obtained from $S_j(\tau)$ using eq. (3). This property is useful for practical analysis.

We then elucidate the physical meaning of $G(\tau)$ distribution as follows. When the statistical weight $g(\tau) \propto 1/\tau$, $1/f$ spectrum can be obtained. McWhorter had proposed a physical model for $g(\tau) \propto 1/\tau$ on the semiconductor surface or in the oxide having the traps [10]. If the carriers in the conducting channel reach the traps by tunneling, the statistical weight $g(\tau)$ can be derived as [20,21]:

$$g(\tau) \propto \frac{dN}{d\tau} = \frac{dN}{dx}\frac{\lambda}{\tau}, \quad (4)$$

where, N is the number of traps (or recombination center), λ is the effective penetration depth and x is the distance from the interface. The uniform distribution of traps (dN/dx = constant) gives the required statistical weight $g(\tau) \propto 1/\tau$ resulting in $1/f$ spectrum. From $G(\tau) = \tau g(\tau)$, we now obtain the relationship $G(\tau) \propto dN/dx$, indicating that $G(\tau)$ is involved in the trap distribution. It should be mentioned that the discussion above is also applicable to the trap distribution in energy assuming the thermal activation of carriers, because the relaxation time depends on the energy in a similar manner as it depends on the distance, such as $\tau \propto \exp[E_A/(kT)]$, where E_A, k, and T are the activation energy, Boltzmann constant and temperature, respectively. Hence, the relationship $g(\tau) \propto 1/\tau$ remains valid even for the thermal activation process, when the traps are uniformly distributed in energy. As of the case of interacting traps [22, 23], as long as the linearity in equations (1) and (2) holds, the proposed model remains valid. Moreover, in a previous work [24], we showed that interacting traps affects the shape of the $G(\tau)$ distribution. It is likely that the $G(\tau)$ contains the information of interacting traps. In order to correctly obtain the information regarding the interaction, it is necessary to include time domain analysis.

Prior to analyzing the experimentally obtained spectrum, we verify the validity of our method by applying it to the two basic spectrum functions: power law function of $S_1(\omega) = c/\omega^{\alpha}$ and $S_2(\omega) = c\tau_c/[1+(\omega\tau_c)^{\beta}]$, where c is a constant. An exact analytical solution of $G(\tau)$ for the former function can be obtained,

$$G(\tau) = \frac{2c\sin(\pi\alpha/2)}{\pi}\frac{1}{\tau^{1-\alpha}}. \tag{5}$$

It is found that $G(\tau)$ has a power-law distribution. This result reproduces a well-known relationship between $1/f^{\alpha}$ spectrum and power-law distribution in McWhorter model [17,18]. It is also found that $G(\tau)$ shows a uniform distribution when $\alpha = 1$, that is, the $1/f$ spectrum case. On the other hand, the latter function $S(\omega) = c\tau_c/[1+(\omega\tau_c)^{\beta}]$, where $\tau_c = 1/(2\pi f_c)$ and f_c is corner

frequency, reproduces the experimentally-obtained power spectra of the low-frequency drain current noise in SiN_x insulator-gate GaAs-based etched nanowire FETs [19,24]. The effective time constant τ_c and the slope of the power spectrum β were successfully evaluated by using this relatively simple and flexible function for $1 \leq \beta \leq 2$. The exact analytical solution of $G(\tau)$ can be derived as:

$$G(\tau) = \frac{c\tau_c \sin(\pi\beta/2)}{\pi} \frac{1}{\tau\{\cosh[\beta \ln(\tau_c/\tau)] + \cos(\pi\beta/2)\}}. \quad (6)$$

The obtained $G(\tau)$ is plotted for various β and τ_c in **Fig. 2**. The coefficient c is 1 in all the calculations. As shown in **Fig. 2(a)**, the distribution changes with β. When $\beta = 1$, $G(\tau)$ shows a uniform distribution for $\tau < \tau_c$, corresponding the $1/f$ spectrum in McWhorter model. When $\tau \rightarrow 0$ with $\beta = 1$, $G(\tau)$ approaches $2c/\pi$. The origin of the $1/f^2$ noise is already known as the random charging and discharging of the discrete electron traps [9]. The $1/f^\beta$ ($1 < \beta < 2$) noise is qualitatively understood as the aggregate of such discrete charging and discharging events with various time constants τ. When $\beta \rightarrow 2$, $G(\tau)$ approaches a δ function such as $G(\tau) = c\tau\delta(\tau - \tau_c)$ whose peak position corresponds to τ_c. For intermediate values of β, the time constant of the dominant event can be readily identified from the peak position. This is exemplified in **Fig. 2(b)** which shows that $G(\tau)$ retains its characteristic shape while its peak position shifts when τ_c is varied.

We apply our method to obtain $G(\tau)$ from the measured power spectra of the drain current noise $S_{IDS}(f)$ given in **Fig. 1(b)**. **Figure 3(a)** shows the calculated $G(\tau)$ distribution from the measured noise spectra. In case of the photo-excitation, we consider that the activation energy is just offset by the photon energy [25] and the relationship of $G(\tau) = \tau g(\tau)$ is not significantly changed, rendering our method still applicable even to this case.

$G(\tau)$ has a narrow peak whose position systematically shifts from 3.4 ms to 88 μs and whose intensity increases with increasing LED light power. This corresponds to the shifting of

the shoulder in the noise spectrum, which becomes more pronounced and simultaneously shifts from 50 Hz to 1800 Hz as the light power is increased as shown in **Fig. 1(b)**. Generally, $1/f$ noise is dominant in the Schottky-gate GaAs-based nanowire FET [26]. However, the calculated $G(\tau)$ for the nanowire FET with TPP molecules shows a very sharp peak, suggesting the random telegraph signal (RTS) noise: the charging and discharging occurs stochastically through discrete states in the molecules. The light-induced drain current noise is attributed to the random sequence of the electron-hole pair generation in TPP by photon supply and its relaxation.

Peak height of the relaxation time distribution $G(\tau_m)$ and the inverse of the peak position $1/\tau_m$ are plotted in **Fig. 3(b)** as a function of the square root of the LED light power, $\sqrt{P_{LED}}$. Both $G(\tau_m)$ and $1/\tau_m$, which correspond to the corner frequency, are proportional to $\sqrt{P_{LED}}$. Our analytical method makes it possible to obtain such relationships without assuming an analytical function for $G(\tau)$ and without rigorous parameter fitting procedure as in the previous analysis performed in Ref. 19. Finally, we briefly discuss the reason why the shoulder in the noise spectrum and its position are affected by light irradiation. Our previous study experimentally verified that TPP only in the metal gate periphery contributed to the observed RTS noise [16]. Taking into account that TPP in the metal gate periphery selectively reflected the RTS noise, a possible reason for $G(\tau_m) \propto \sqrt{P_{LED}}$ and $1/\tau_m \propto \sqrt{P_{LED}}$ is qualitatively explained as follow. When the charge relaxation time is constant, the RTS noise power P_{RTS} and $1/\tau_m$ are proportional to the mean pulse generation rate in the drain current. Because one photon generated one current pulse via single TPP excitation, P_{RTS} and $1/\tau_m$ linearly depend on the photon supply rate given by the number of the photons per second. Considering that TPP only in the metal gate periphery contributes to the RTS noise and the photon supply rate for the one-dimensional gate edge is of the order of $\sqrt{P_{LED}}$, P_{RTS} and $1/\tau_m$ are proportional to $\sqrt{P_{LED}}$.

Computing for P_{RTS} by integrating power spectrum density $S(\omega)$ from $\omega = 0$ to ∞, we find that P_{RTS} is proportional to $G(\tau_m)$. Hence, $G(\tau_m)$ is also proportional to $\sqrt{P_{LED}}$.

In summary, to derive the relaxation time distribution of the dynamic charge events in a transistor from the current noise spectrum, we developed an analytical method applying an inverse integral transformation method to the McWhorter model. We demonstrated the feasibility of our method by characterization of the charge dynamics in a GaAs-based nanowire FET with molecules dispersed on its surface.

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Appendix

The basic idea of the method applied in solving the integral equation has been described by well-known Cole-Cole, Davidson-Cole and Havriliak-Negami models and their relaxation time distribution functions in the field of dielectric relaxation of polymers [27]. Transformation of variables given below

$$p = \omega^2, \quad q = 1/\tau^2, \quad M(p) = S\left(p^{1/2}\right), \quad h(q) = \frac{1}{2} q^{-1/2} G\left(q^{-1/2}\right) \tag{A1}$$

reduces Eq. (2) to the form

$$M(p) = \int_0^\infty \frac{h(q)}{p+q} dq \,. \tag{A2}$$

Equation (A2) shows that $M(p)$ is in the form of double Laplace transform (also called as Stieltjes transform) of $h(q)$. Accordingly, we can obtain the generalized $G(\tau)$ distribution given as Eq. (3) in the main body of the manuscript.

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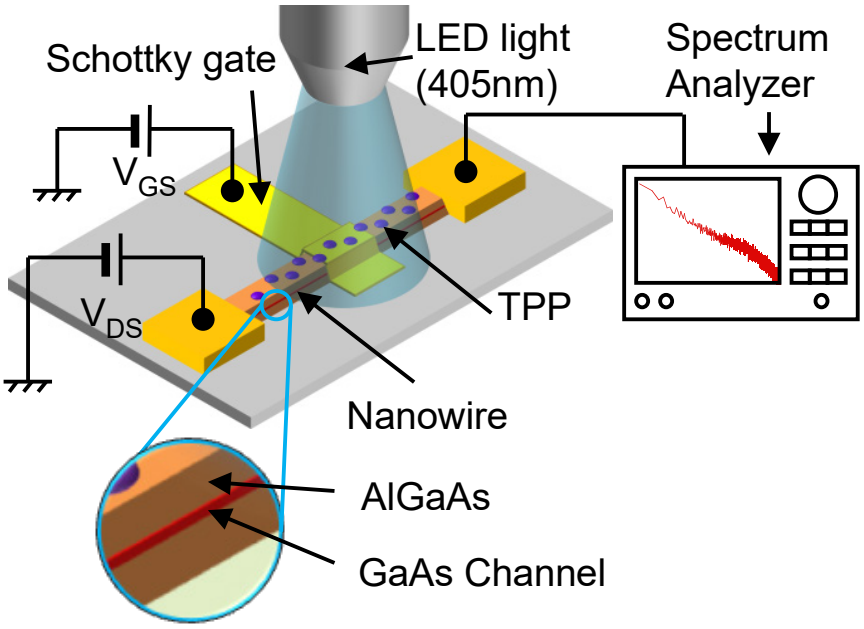
Figure captions

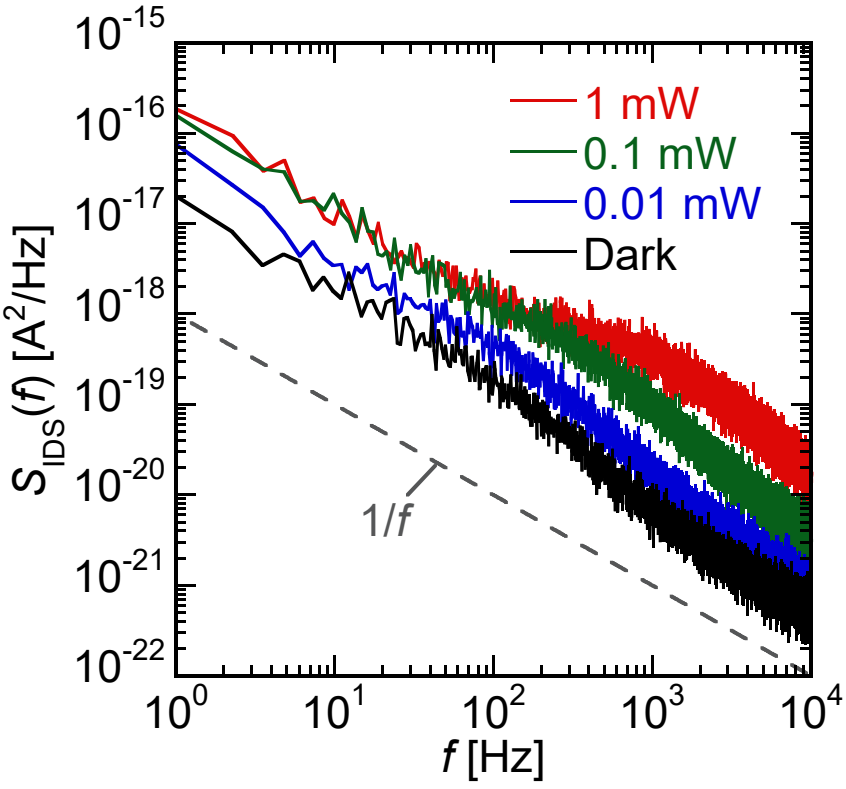
Fig. 1 (a) Schematic illustration of a GaAs-based nanowire FET with TPP dispersed on the surface. **(b)** Measured drain current noise spectra at V_{DS} = 0.8 V and V_G = 0.5 V under light irradiation and dark condition.

Fig. 2 $G(\tau)$ distribution obtained using Eq. (6) for **(a)** several values of noise exponent β at a fixed time constant τ_c = 1 s and **(b)** several values of time constant τ_c at a fixed noise exponent β = 1.5.

Fig. 3 (a) Calculated $G(\tau)$ distribution obtained by using the general expression of $G(\tau)$ given in Eq. (3) and **(b)** evaluated peak intensity $G(\tau_m)$ and inverse of their position $1/\tau_m$ plotted against the root of incident LED irradiation power $\sqrt{P_{LED}}$.


Schottky gate
LED light
(405nm)
Spectrum
Analyzer
V_{GS}
V_{DS}
TPP
Nanowire
AlGaAs
GaAs Channel

(a)


1 mW
0.1 mW
0.01 mW
Dark
1/f
$S_{IDS}(f)$ [A^2/Hz]
f [Hz]
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^0
10^1
10^2
10^3
10^4

(b)

Figure1

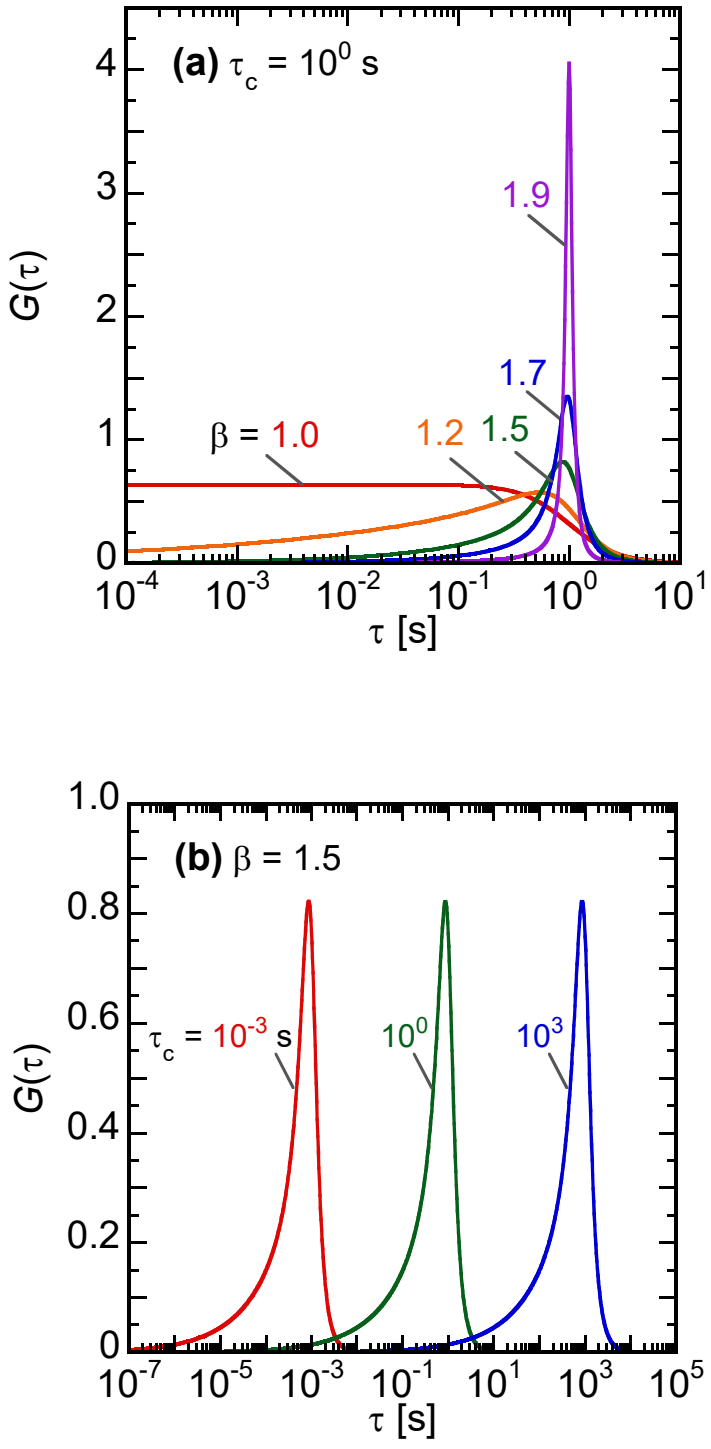


Figure2

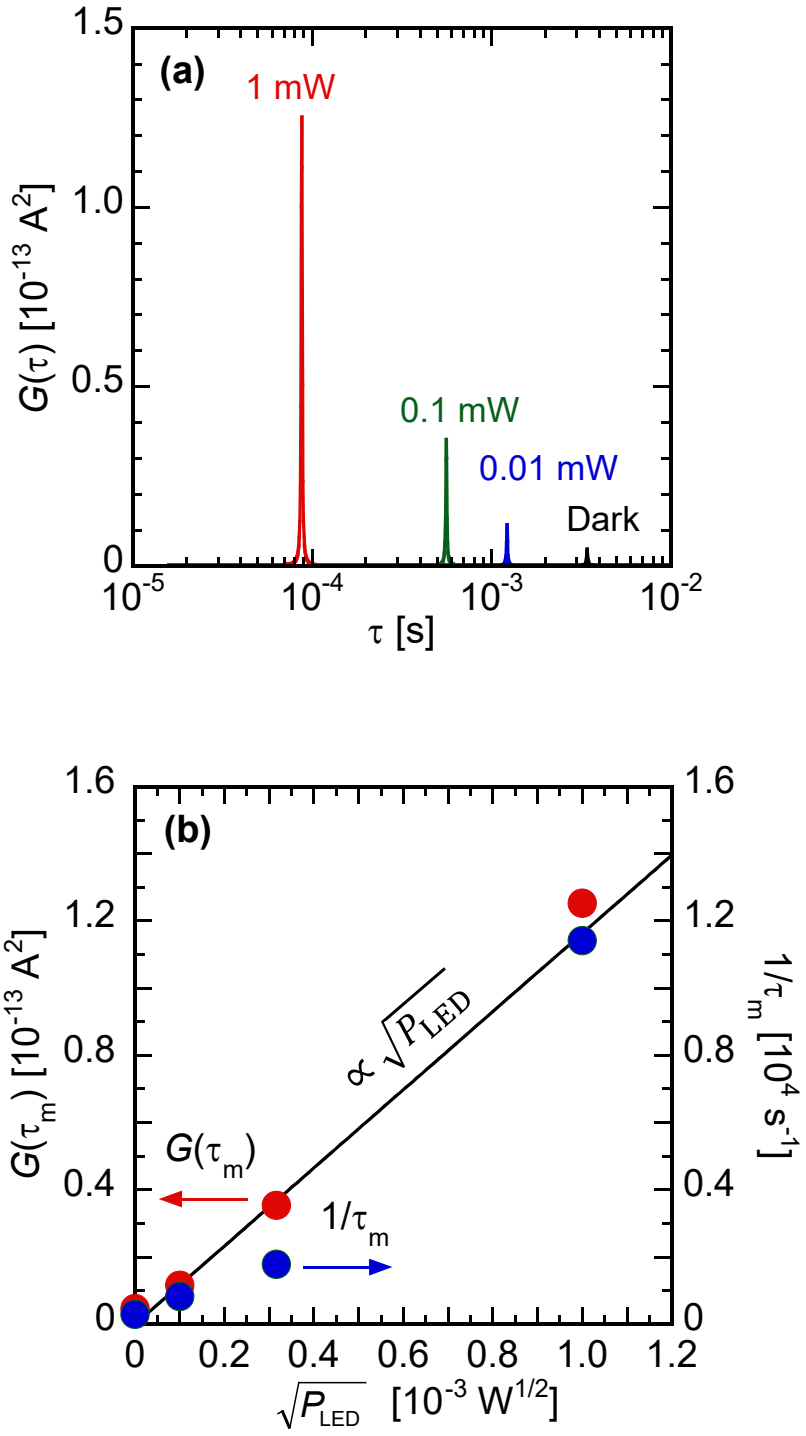

(a)
1 mW
0.1 mW
0.01 mW
Dark
G(τ) [10^-13 A^2]
τ [s]
(b)
G(τ_m) [10^-13 A^2]
1/τ_m [10^4 s^-1]
∝ √P_LED
G(τ_m)
1/τ_m
√P_LED [10^-3 W^1/2]

Figure3

Supplementary data for "Analytical derivation of charge relaxation time distribution in a transistor from current noise spectrum using inverse integral transformation method"

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