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Authentic and Inauthentic Number Representations in Husserl's *Philosophy of Arithmetic*

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Abstract: This paper aims to elucidate the grounds for the distinction between authentic and inauthentic representations in Husserl's *Philosophy of Arithmetic* by considering their central characteristics. The distinction between these two sorts of representation is introduced at the very beginning of the work. Yet the grounds for it are hardly given there. Thus, it may initially seem that the distinction is groundless. However, as we move on to the second part of the book, Husserl accounts for the distinction by mentioning our cognitive incapacity. This incapacity is pertinent to a limit of our number representation: in short, we cannot represent all numbers authentically. Such a limit of authentic number representation is made apparent by reference to the representation itself. Thus, it can be said that Husserl's detailed discussion of authentic number representation in the first part of the work is vital to the very ground for the distinction. It goes likewise for his discussion of the inauthentic sort in the second part. To show this point and to consequently give the grounds for the distinction, I make an overview of the central characteristics of both sorts of number representation. At the end of the paper, I conclude with a brief remark on two possible further interpretations of the work: what Husserl means by the logical equivalence between authentic and inauthentic number representations, and what kind of bearing Husserl's characterization of inauthentic number representation has on the concept of infinity.

Introduction

At the basis of Husserl's *Philosophy of Arithmetic*,[1] there lies the fundamental distinction between authentic and inauthentic number representations. The distinction is so fundamental to the work that without it, the structure of the work would be entirely different; the first part mostly deals with the authentic one, and the second with the inauthentic counterpart. However, at the beginning of the work, the grounds for the distinction are barely given. It may therefore seem that Husserl almost takes it for granted.

It is only when he sets about discussing the inauthentic number representation in the second part of the work that Husserl provides us with the grounds for the distinction. To note it briefly, the grounds are related to our factual cognitive incapacity. It thus appears that Husserl does not introduce the distinction as

1 When referencing the work, I put PA and a page number from *Husserliana* Band XII. All translations of the direct quotes are made by the author of this paper. Supplements by the author are made with the square brackets.

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something unrelated to our activity. On the contrary, the distinction has its basis in our practice of authentically and inauthentically representing numbers.

The primary goal of this paper is a concise exposition of the central characteristics of authentic and inauthentic number representations in Husserl's *Philosophy of Arithmetic*. This would, if successful, not only give such an exposition but also present us with a possible reason why Husserl did not initially give grounds for the distinction. Put differently, by picking up the central characteristics of each representation, I attempt to point out that the distinction is grounded in itself, and that Husserl, from the beginning, immediately looks into the authentic number representation so that it is grounded in itself.

I structure this paper as follows. In the first section, the central characteristics of authentic number representation are delineated. The characteristics here are the limit and the activity when numbers are authentically represented. Second, I summarize the main characteristics of inauthentic number representation. For this sort of representation, I pick up several inauthentic ways of number representation and the relationship between arithmetic and inauthentic number representation. In the concluding part, based on the preceding exposition of two sorts of number representation, I note two things: a possible reason why Husserl did not outright give grounds for the distinction at the beginning of the work, and further possible investigations that pertain to the interpretation of *Philosophy of Arithmetic*.

Authentic Number Representation

The first sort of number representation that will be dealt with is the authentic one. This authentic sort, being the main theme of the first part of *Philosophy of Arithmetic*, can be said to be more fundamental than the inauthentic counterpart. This is so because inauthentically represented numbers "presuppose" (voraussetzen) the authentically represented ones (PA 16).[2] The most straightforward characteristic of them is their directness. It must, however, be added that this is too vague a characteristic, and it is unclear how precisely it is direct. Even in contrast to the inauthentic counterpart, which will be dealt with in the next section, the directness of the authentic sort requires further specification by itself. Husserl's specification of directness is broadly twofold: he refers to the experience of authentically representing and the numerically limited thereof. In the following, I expound upon these two.

Husserl's exposition of the authentically represented numbers is first limited to what he calls a "multiplicity" (Vielheit). To be precise, a multiplicity is not an authentically represented number. It is nonetheless the case that whenever a number is represented, a multiplicity is also represented (PA 14-15). The difference between the two is their determinateness: if a multiplicity is determinate as to its "how many?" then, it is an authentically represented number. Thus, the activity that is required for the representation of a multiplicity should also be held in the case of authentic number representation. Hence, Husserl is first concerned with the activity that enables the representation of a multiplicity. There, Husserl, by comparing it with other enabling conditions such as time, comes to say that the collecting activity is the basis of representing a multiplicity (PA 30). This collecting activity combines each object that is collected,

2 Although I do not here delve into the presupposition relation holding between the two sorts of number representation, this relation requires special attention. It seems conceivable that someone first comes to have inauthentically represented numbers, such as the Arabic numeral nine. Thus, it may seem that an authentically represented number is not necessarily presupposed when having an inauthentically represented one. Husserl does not discuss this point, but at the same time, Husserl does not say that this presupposition relation holds only within an individual.

and what he terms "collective combination" (kollektive Verbindung) comes to hold within a multiplicity.

Given that there is a collecting activity that enables our representation of a (determinable, and thus numerically representable) multiplicity as a group of collectively combined objects, we can now proceed to the discussion of what precisely such groups are. One frequently-mentioned characteristic of authentically representing a number is our incapacity to do so for all numbers: Husserl points out that, for example, we can authentically represent only the first few numbers (PA 190). From this, it follows that Husserl had in mind that authentic number representation and the numbers thereof have their limit. Two main points can be discussed in relation to this limit. First, this limit Husserl associates with the authentic way is not so determinate for Husserl himself; at most, it is approximately twelve (PA 192). Followingly, the precise determination of the maximal authentically representable number is not of much importance. Another point is that there seems to be no minute discussion of the limit of authentic number representation. This may at first glance seem rather cryptic, for without a determinate limit of the authentically representable number, the distinction cannot be made in an extensionally determinate way from the inauthentic counterpart: we cannot determine from where the inauthentically represented number series starts. If one were to take the distinction fully extensionally, clarification on the precise limit of authentic number representation would be necessary. This is because, if the limit is not determined at all, it could be that there is no such thing as a limit. However, as we will see in the next section, what is crucial to the distinction is there being at least a certain limit to authentic number representation, and the inauthentic counterpart supplementing it. Consequently, the precise determination of such a number is not necessary in all respects.

The possible interpretation of this indeterminateness can be, in addition to its peripheral importance for the distinction, accounted for by reference to Husserl's characterization of arithmetic as mostly inauthentic. It must be noted that, for Husserl, arithmetic hardly operates with authentically represented numbers: he goes so far as to say that "[...] all number representations that we have beyond the first few in the number series are symbolic [that is, inauthentic], and can only be symbolic; a fact which completely determines the character, sense, and purpose of arithmetic" (PA 190). Here, the practice of arithmetic already serves to show that beyond some authentically representable number, there are numbers that are only inauthentically representable. We may therefore conclude that authentic number representation cannot proceed to so great an extent that it covers all numbers.

Above, we have seen the fundamental activity that enables authentic number representation, namely, that of collecting, and the objects represented there. Given that these characteristics are noted by reference to the representation itself, it can be summed up that the authentic one is grounded in itself.

Inauthentic Number Representation

Provided that the authentic number representation is thematized, it may seem that the inauthentic counterpart, which could just be a negation of authenticity, does not require any reference in our experience. For Husserl, notwithstanding, direct reference to the inauthentic one is no less important than that of the authentic: inauthentic number representation is the focal point of the second part of *Philosophy of Arithmetic*. While the authentic number representation is direct, the inauthentic sort is indirect. In this section, I delineate two main characteristics of it: its several ways of representing numbers, and its peculiar relation to arithmetic.

First, the most straightforward characteristic of inauthentic number representation is the use of signs

(PA 193). Let us briefly see what Husserl means by signs. When we authentically represent a house, we look at it, whereas when we inauthentically represent it, we rely on a description of the house, the description of which is for the house and nothing else. Here, signs mediate the representation of the house, and what is important is that the house is described in a way that the description stands for the very house and not for others. As Husserl's example shows, signs in number representation are used to satisfy what he calls the "uniqueness of the characterization" (Eindeutigkeit der Charakteristik). This uniqueness is essential to the signs here, and also to the inauthentic number representation that is mediated by signs. To account for such uniqueness, Husserl contrasts inauthentic representation with "general representation" (allgemeine Vorstellung): while we can think of different individuals falling under the concept human, in the inauthentic number representation, there is only one object which falls under an inauthentically represented number (ibid.).[3] Although the general one may seem straightforward, the inauthentic one requires further clarification. What Husserl has in mind by the uniqueness is the exclusion of any number of things except that of "something" (Etwas). That is, the uniqueness here is meant to exclude the conception of number such that, under the inauthentically represented number four, there fall four books, cats, dogs, etc. This conception is not acceptable. It is thus the case that, through the inauthentically represented number four, we are directed to nothing but the group of four somethings.

There are other ways in which numbers are inauthentically represented: the case where inauthentically representing the number of things involves what Husserl calls "sensible group" (sinnliche Menge). The occasions Husserl has in mind when talking about this inauthentic representation are those in which we immediately notice the number of things, despite there being so many of them that we cannot grasp the precise number of them. Let us consider this with the following example: the occasion on which we see a flight of birds. When we see the flight, when there are many, we cannot immediately grasp the number of them. Yet it is still the case that we can grasp that there are some birds. When we grasp that there are some birds, the number we give to the flight is not a determinate one: it remains indeterminate, and only after counting, we come to conclude that there is a specific number of birds. What is peculiar in this example, and the feature that Husserl wants to focus on in it, is our noticing of "there are some …," which is indeterminate. This representation is also what Husserl includes in the inauthentic sort. This "there are some …" is distinguished from our precise number determination of things that we find in the authentic cases: the cases in which the activity of directly collecting is made (PA 195-196).

The indirect grasping is not limited to the indeterminate many. It can also be determinate, yet remain inauthentic. "For example in the game of dominoes, we grasp groups of ten to twelve dots at a glance; indeed, we even recognize their number quite immediately" (PA 28 Anm.). Just like in the case of the indeterminate many, we can immediately notice that many objects are to be counted, but in this case, in a determinate way. However, Husserl continues, what is being the case is that the number name here is "directly associated with the characteristic sensory appearance, and it can now be recalled through it at any time without any conceptual mediation. With quantities of this size, as anyone can verify, a direct and proper collection and counting is an impossibility" (ibid.). This way, Husserl also considers our immediate association of a configuration with a number name. This holds not only for number names.[4]

Provided that inauthentically represented numbers are represented through signs that uniquely refer to

3 Husserl brings these two into contrast just to highlight that inauthentic number representation is done with uniqueness. Thus, a representation's being either inauthentic or general does not mean that it has to be either of them.

each number, we may now turn to Husserl's characterization of arithmetic. As we have briefly seen in the previous section, Husserl thinks that when we speak of arithmetic, we have to consider "[...] the fundamental fact that all number representations that we have beyond the first few in the number series are symbolic, and can only be symbolic; a fact which completely determines the character, sense, and purpose of arithmetic" (PA 190). We may put this in the following way. Arithmetic does indeed presuppose a few authentically represented numbers. And as we also do, we have our authentic activity of collecting. This activity is a combination of the new to the collected (ibid.). Given that we have an authentic operation, we may presume that operations in the whole of arithmetic are also operations in an authentic sense. Yet "what arithmetic calls 'operations' corresponds not at all" (ibid.) to the authentic operations.[5] Moreover, to further his claim that inauthentic number representation is pivotal to arithmetic we have, he takes into account the counterfactual case in which all numbers were authentically represented. To clarify, let us suppose that we could authentically represent all numbers. For Husserl, arithmetic in such a case "would then be completely superfluous" (PA 191). To show this point, he continues as follows:

> The most complicated relations between numbers, which are now discovered only laboriously through circuitous calculations, would then, with the number representations, be simultaneously present to us with the same intuitive evidence as propositions such as 2 + 3 = 5. To anyone who knows what 2, 3, 5, and the signs + and = mean, this proposition is immediately and with evidence clear. (ibid.)

To sum up, we can say that Husserl has a twofold characterization of arithmetic as inauthentic. First, arithmetic in practice is factually inauthentic. This is clear from there being operations and numbers that are inauthentic. "Inside and outside of the science [namely, arithmetic], one speaks as if one could continue the sequence of numbers to infinity, that is, beyond any attained limit" (PA 192), even though we cannot have all numbers authentically represented. Second, he employs a counterfactual case in which we could authentically represent all numbers. This would make arithmetic superfluous, for all complex operations through which we come to have other numbers would be made apparent just for the sake of making them apparent. In such a case, it would at least be the case that operations would not be there for bringing numbers that would have been difficult to achieve without them. Now this may sound possible and acceptable, particularly given that arithmetic without practical orientation is still conceivable. That is, it could be that arithmetic is no more than making apparent the relations of numbers that are not authentically apparent.[6] However, for Husserl, "the whole of arithmetic is, as we will see, nothing other than a sum of technical means for overcoming the essential imperfections of our intellect" (PA192). Followingly, based on the fact that we cannot represent all numbers authentically, the purpose of arithmetic is not about coming up with operations just for itself.

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4 Husserl is well aware of the fact that the limit of such inauthentic grasping of the number of things can vary, such as "through continued and methodical practice" (PA 254).

5 The detailed consideration of this point is central to the second part of the work. Husserl discusses the contents of various operations in arithmetic, and show that they are themselves not the operations in the authentic sense.

6 This line of thought seems possible since Husserl says that the only authentic operations are combination and partition (PA 190). Other operations could still be made apparent had we represented all numbers authentically, and the purpose of arithmetic may lie therein. This way of thinking, however, is not conclusive at all, for even those operations (which are made apparent when representing all numbers authentically) may still be made apparent for the sake of attaining numbers, all of which are here already supposed authentically represented. This line of thought is presumably not aligned with that of Husserl's in *Philosophy of Arithmetic*, given that Husserl mentions arithmetic's "practical" aspect in several places (PA 192, 245).

To summarize, inauthentic number representation is broadly characterized by its indirectness. To such a central feature, Husserl gives various ways it is so, all of which can be noticed by reference to our practice or experience. Thus, the inauthentic sort of number representation is also something that is grounded in itself, making the distinction not arbitrary at all.

Conclusion

In this paper, I explicated the main characteristics of authentic and inauthentic number representations. Both of them are grounded in themselves. By way of conclusion, I briefly note that, precisely because both representations can be found in our experience, the distinction in *Philosophy of Arithmetic* is grounded by the very discussion of each. Here, we can presume that the distinction was not something that was going to be used by being set arbitrarily, but was something that was going to be explicated by direct reference to each.

This fundamental distinction being clarified, there are two possible further interpretations of the work. First, given this distinction, Husserl claims that numbers under the distinction are in the relation of "logical equivalence" (logische Äquivalenz). Husserl does not delve into this relation so much in the work.[7] This relation seems, however, vital to arithmetic: if the two representations were not directed towards the same number, then arithmetic, that is largely inauthentic, would not stand for the same numbers we authentically represent. Another consideration could be that of the infinite within the range of inauthentic number representation. As we have seen above, the authentically representable numbers are more limited than the inauthentic ones. While this is the case, Husserl considers the limit of numbers that are representable in inauthentic ways. An example is, as we saw in the previous section, the limit in the case of the immediate association of a number with a configuration. The case of the infinite, which could be said to be one of the extreme cases of inauthentic number representation, is not extensively touched upon. The discussion of the infinite is of importance, particularly given that the use of the infinite was then one of the central concerns of the foundation of mathematics.

Reference

Husserliana Bd. XII: *Philosophie der Arithmetik*. Mit ergänzenden Texten (1890-1901), Den Haag: Martinus Nijhoff, hrsg. von L. Eley, 1970.

7 His claim on this relation is kept rather minimal: "I note that the authentic representation and a symbolic representation matching it stand in the relation of logical equivalence. Two concepts are logically equivalent when each object of the one is also an object of the other, and conversely" (PA 194). This extensional account by Husserl is unclear in two regards: to which concepts the logical equivalence holds, and whether the relation holds. In short, the relationship among objects, representations, and concepts is not so obvious.