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Erosion Mechanisms of Baffle Plates at the Stagnation Point in Hybrid Rockets

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In order to use hybrid rockets as kick motors, it is essential to address their inherently low combustion efficiency issue. While baffle plates are employed to improve combustion efficiency, their structural failure due to erosion phenomena caused by combustion gases becomes a serious concern. This study aims to identify the dominant parameters governing the erosion rate at the stagnation point of an impingement flow and to establish a predictive numerical model. Experiments were conducted using an HDPE/N₂O hybrid rocket, varying the impinging jet velocity gradient and chamber pressure. A predictive model for the baffle plate erosion in hybrid rocket environments was developed by modifying the heterogeneous reaction mechanism in a preexisting theoretical model established in the field of carbon combustion. The experimental results revealed that the velocity gradient exponent was lower than the theoretical value under diffusion-limited conditions. Also, a dependence of the baffle plate erosion rate on the surface temperature was observed, indicating the erosion rate is not diffusion-limited. The application of the theoretical model demonstrates good alignment between predicted and experimental results with an average error of about 33 % without any corrections. This confirms the applicability of the theoretical model for predicting baffle plate erosion in hybrid rockets.

Nomenclature

(Nomenclature entries should have the units identified)

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a	=	velocity gradient, 1/s
A	=	area, m ²
$A_{i,s}$	=	Damköhler number
B_i	=	frequency factor, m/s
C^*	=	characteristic velocity, m/s
C_p	=	specific heat, J/kg·K
D	=	diameter, m
D'	=	diffusion coefficient, m/s
E_i	=	activation energy, kJ/mol
f_s	=	stream function at a surface
G	=	mass flux, kg/m ² ·s
h	=	heat transfer coefficient, W/ m ² ·K
H	=	Height, m
k	=	thermal conductivity, W /m·K
k_i	=	specific rate constant of i -th reaction
K	=	correction factor for regression rate
K'	=	correction factor for mass transfer coefficient
ΔL_{er}	=	erosion amount, m
L_f	=	fuel length, m
MW	=	molecular weight, g/mol
$\dot{m}$	=	mass flow rate, kg/s
P	=	pressure, Pa
Pr	=	Prandtl number
$\dot{q}$	=	heat flux, W/m ²
$\dot{r}$	=	fuel regression rate, mm/s
R	=	ideal gas constant, J/ mol·K
Re	=	Reynolds number
t	=	time, s

Δt	=	timestep, s
T	=	temperature, K
V	=	velocity, m/s
Y	=	mass fraction

Greek Symbols

β	=	Spalding transfer number
γ	=	specific heat ratio
δ	=	boundary layer thickness, m
η	=	combustion efficiency
μ	=	viscosity, kg/m·s
ξ	=	profile function
ρ	=	density, kg/m ³
ω	=	mass ratio of CO ₂ and C
Ψ	=	stream function

Greek Symbols

$\sim$	=	stoichiometric mass weighted
$-$	=	average

Subscripts

aft	=	aft-chamber
b	=	burn duration
BF	=	baffle plate
c	=	chamber
ex	=	experiment
er	=	erosion
f	=	fuel
g	=	combustion gas
s	=	surface
t	=	throat

th = theory

∞ = free stream

I. Introduction

IN the recent past, hybrid rockets have attracted much attention in the field of propulsion systems for small satellites [1] and launch vehicles[2]. Chemical rockets are generally classified into three major categories: solid rockets, liquid rockets, and hybrid rockets. Compared to solid and liquid rockets, hybrid rockets possess attractive safety characteristics, such as inherent non-flammability and non-explosivity due to the separation of fuel and oxidizer into distinct phases and the common use of non-toxic substances. Owing to their superior safety profile, hybrid rockets not only reduce handling costs[3] but also render them suitable for rideshare space vehicles, which are launched together with other spacecraft and must minimize explosion risks. Furthermore, hybrid rockets exhibit numerous desirable characteristics such as thrust controllability, long-duration storability, a simple feed system, and restart ability. On the other hand, in order to improve the competitiveness of hybrid rockets, it is necessary to address the low combustion efficiency issue intrinsically linked to their combustion process [4]. In general, the combustion process in single-port hybrid rockets involves the formation of a diffusion flame near the inner surface of the solid fuel. From the wall to the flame region, the combustion reaction becomes fuel-rich, while above the flame zone, it becomes oxidizer-rich. As a result, it is difficult to achieve efficient mixing and complete combustion of the fuel and oxidizer, which leads to low energy obtained from combustion.

In order to improve the combustion efficiency in hybrid rockets, many researchers have studied a mixing device known as a baffle plate[5–7] as shown in Fig. 1. The baffle plate enhances the mixing through the creation of impingement flow on its upper surface when it is inserted in the middle or bottom of the fuel. Wada et al. [6] examined the relationship between the combustion efficiency and the characteristic length, both with and without a baffle plate. They found that the combustion efficiency remains high even if the characteristic length is small. This feature is suitable especially for the kick motor because its volume is heavily restricted.

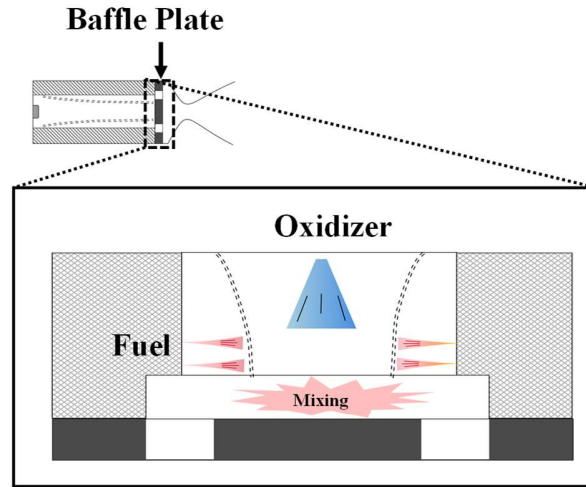


Fig. 1 Schematic diagram of a mixing enhancement by a baffle plate

There is a significant challenge in employing a baffle plate due to the failure issue arising from the erosion phenomenon. Typically, baffle plates are made of graphite known for their high heat resistance. Whereas graphite can withstand high temperatures, it is prone to erosion by the chemical reaction with the oxidizing species in combustion gas. According to Hill et al., in several experiments, they suffered baffle plate failure within approximately five to seven seconds due to the erosion[5]. These failures not only reduce combustion efficiency but also, looking to real in-space applications, contribute to the generation of solid debris. Despite these critical issues, there have been few experimental data and literature studies about erosion rates at the stagnation point in impingement flow in hybrid rockets. In the field of hybrid rockets, the graphite erosion phenomenon has been studied in application as ablative thermal protection devices for nozzles. Nasuti et al. [8] considered O_2 , H_2O , CO_2 , OH and O as the primary oxide species and predicted the erosion rate numerically. Although the chemical reaction mechanisms may be similar between nozzle erosion and baffle plate erosion, the species diffusion rate differs across the chemical boundary layer due to the parallel flow at the nozzle throat compared to the impingement flow at the baffle plate. Consequently, the dependencies of the graphite erosion process on key flow-related parameters, i.e., flow pressure and velocity, will be different in the case of the baffle plate if compared with the more well-established nozzle application. In the context of carbon oxidation in impingement flow, extensive research has been conducted in the field of carbon combustion. Walker et al. [9], in a comprehensive review, stated that the carbon combustion phenomena are mainly controlled by the chemical reaction rate or the diffusion rate. Based on the experimental results of Day et al.[10], in which the

combustion rate was proportional to the square root of the velocity of the oxidizer, they explained the erosion rate was dominated by the diffusion rate in the high temperature regime. Makino et al. [11] constructed a prediction model for the carbon combustion rate by integrating the chemical reaction and the diffusion phenomena at the carbon surface. Although this model was able to reconstruct the experimental results fairly, two issues arise when applying it to hybrid rockets: 1) the oxidizer species considered in the reaction mechanism at the surface are only O_2 , H_2O and CO_2 , and 2) all experiments were conducted under atmospheric pressure conditions. According to Rotondi et al, under stoichiometric conditions, OH has a contribution of 18% of the overall erosion rate of the nozzle in typical hybrid rocket[12]. Therefore, to adopt the model for the erosion rate in hybrid rockets, it is necessary to include additional chemical species. Moreover, a verification of the model against experimental data obtained from hybrid rockets has to be carried out.

This study aims to clarify the parameter dominating thermochemical erosive phenomena at the stagnation point of an impingement flow in a graphite-based baffle plate employed in hybrid rocket environment. By focusing on the erosion phenomenon at the stagnation point of an impingement jet, rather than the conventional nozzle throat region, this research not only contributes to the development of a predictive model for the baffle plate's erosion rate but also deepens the fundamental understanding of erosion mechanisms, particularly in relation to species diffusion effects in hybrid rockets. According to Makino et al. [11], the velocity gradient is the single parameter that characterizes the flow field in the impingement flow. Therefore, combustion experiments were conducted under varying velocity gradients. Furthermore, to elucidate the influence of pressure on the erosion rate, additional experiments were performed at different chamber pressures. Additionally, a prediction model, derived from expanding the original Makino's model, was developed and compared with the experimental results.

II. Method

A. Experiment

1. Experimental setup

All experiments were conducted using a 5N-class hybrid rocket motor, with high-density polyethylene (HDPE; formula C_2H_4) as a fuel and nitrous oxide as an oxidizer, which is considered the most suitable choice for kick motors compared to conventional oxidizers [1]. A schematic diagram of the experimental apparatus is shown in Fig. 2. Ignition was initiated by energizing nichrome wires with a 24 V battery for ten seconds, followed by a two-second

supply of gaseous oxygen. Subsequently, gaseous nitrous oxide was introduced for 15 seconds after ignition. A cross-sectional view of the combustor is illustrated in Fig. 3. The combustor consists of HDPE fuel, an aft-chamber, a baffle plate, and a nozzle. The aft-chamber, the baffle plate, and the nozzle were made of the same grade of isotropic graphite (ET-10 provided by Ibiden Co. Japan). The baffle plate used in the experiment featured four holes, each with a diameter of 3.0 mm. The hole diameters were designed such that the total cross-sectional area of the baffle plate was three times that of the nozzle throat, following the methodology described by Hill et al. [5]. The baffle plate thickness was set to 8 mm, based on Makino's experimental findings showing a maximum erosion rate of 0.032 mm/s[11], to ensure sufficient structural margin against the expected erosion during a 15 s burn. The pressure upstream of the baffle plate was measured through the hole drilled in the aft-chamber using a KYOWA DCS-5 MPa pressure sensor with rated accuracies of ± 0.01 MPa. The baffle plate temperature was measured from the hole drilled 4.0 mm below the upper surface by using an RC Pro k-type thermocouple. All pressure sensors and thermocouples were connected to KYOWA EDX-100 and recorded at a frequency of 200 Hz.

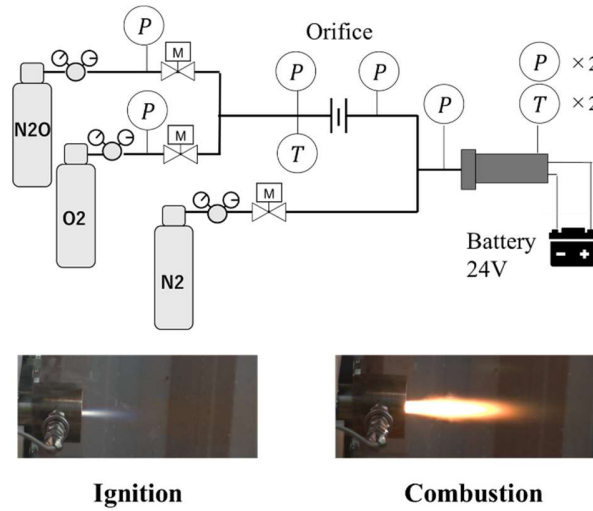


Fig. 2 Schematic diagram of the experimental apparatus

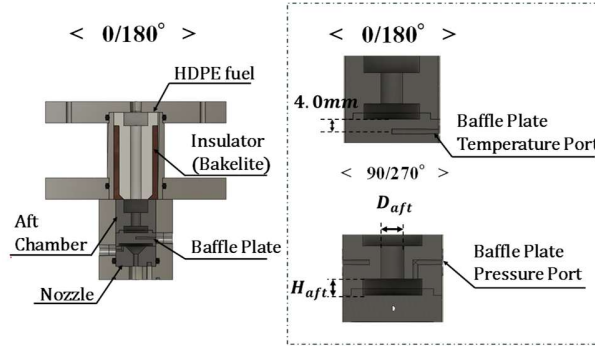


Fig. 3 Cross-section diagram of the combustor employed in this study

The experiments were conducted in two distinct series. The first, referred to as the Velocity Gradient series, involved varying the velocity gradient with the same nozzle throat diameter and oxidizer mass flow rate. In particular, the impingement flow velocity was modulated by varying the aft chamber diameter D_{aft} (see bottom-right of Fig. 3). Additionally, the distance between the aft chamber exit and the surface of the baffle plate H_{aft} was varied, which resulted in full control of the flow velocity gradient. When varying the flow velocity by adjusting the aft chamber diameter, potential variability in velocity was taken into account; accordingly, for each geometric configuration, a minimum of three experiments were conducted under identical conditions. The second series, named the Chamber Pressure series, involved changing the gas pressure by using different nozzle throat diameters while maintaining a constant oxidizer mass flow rate. To ensure the same velocity gradients across different gas pressures, the aft-chamber diameter was adjusted.

2. Measurement method of the erosion amount

A 3D scanner (VL-700: KEYENCE Co.) was used to evaluate the erosion amount of the upper surface of a baffle plate. Figure 4 illustrates the measurement procedure for determining the erosion amount. The shape of the baffle plate was measured both before and after each experiment with an accuracy of 0.01 mm. The two 3D models were aligned by matching the position of the protrusion's top surface and the phase of the thermocouple hole. Zero degrees was defined as the phase where the centers of two holes were aligned, and cross-sections were extracted at every 45-degree interval. In each cross-section, the midline of the outer wall of the baffle plate was determined. Lines were drawn from the midline at intervals of 1.0 mm up to 2.0 mm. The distances between the baffle plate profile before and after the experiments were determined as the local erosion amounts. Therefore, each cross-section yielded five data points representing the local erosion amounts. The experimental erosion amount was defined as the average of 20 local erosion measurements.

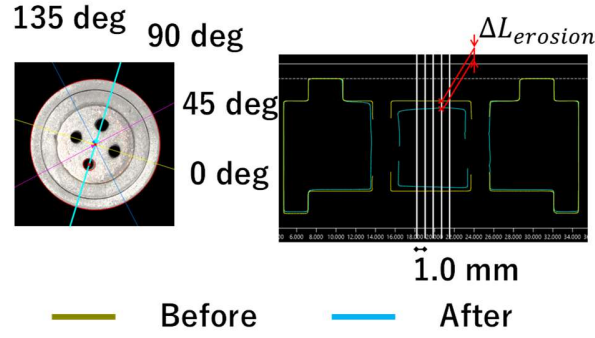


Fig. 4 A procedure for measuring erosion amounts

3. Data arrangement

The erosion rate was expressed as a mass flux in the same way as Makino et al.[11] and calculated from Eq. (1). The graphite density was $1740 \pm 40 \text{ kg/m}^3$, defined as the ratio of the baffle plate mass to the volume obtained from the 3D CAD model.

$$\dot{m}_{er} = \frac{\rho_g \Delta L_{er}}{t_b} \quad (1)$$

Assuming that the axial pressure gradient is negligible, the velocity gradient in the axisymmetric stagnation flow field was expressed as Eq. (2). To calculate the gas velocity, a uniform flow within the aft-chamber was assumed.

$$a = \frac{V}{H_{aft}} = \frac{\dot{m}_f + \dot{m}_{ox}}{\rho_c A_{aft} H_{aft}} \quad (2)$$

Gas density was determined using Eq. (3).

$$\rho_c = \frac{\overline{P_{BF}}}{R/MW T_{c,ex}} \quad (3)$$

To evaluate realistic gas properties, the combustion gas temperature was modified using combustion efficiency, which was calculated from Eqs. (4) and (5). The molecular weight and the theoretical characteristic velocity (C_{th}^*) were determined by the NASA CEA code[13].

$$C_{ex}^* = \frac{P_c A_t}{\dot{m}_f + \dot{m}_{ox}} \quad (4)$$

$$\eta_{c^*} = \frac{C_{ex}^*}{C_{th}^*} \quad (5)$$

As shown in Eq. (6), C_{th}^* is proportional to the square root of combustion gas temperature [14]. Therefore, the experimental gas temperature was modified by multiplying the squared combustion efficiency factor.

$$C_{th}^* = \frac{\sqrt{\gamma R T_c}}{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad (6)$$

$$T_{c,ex} = \eta_{c,*}^2 T_{c,th} \quad (7)$$

B. Theoretical model to predict the erosion rate

The calculation flowchart for predicting the erosion rate used in this paper is illustrated in Fig. 5. The calculation process is divided into three steps. Initially, the combustion efficiency and O/F ratio history were determined using the fuel regression rate formula and experimental data to obtain the gas temperature history by using Eq. (7). Based on these results, a one-dimensional heat transfer analysis was performed to ascertain the surface temperature of the baffle plate. Finally, the erosion rate of the baffle plate was calculated from the theoretical model derived from the field of carbon combustion. The subsequent sections provide a detailed explanation of each computational method.

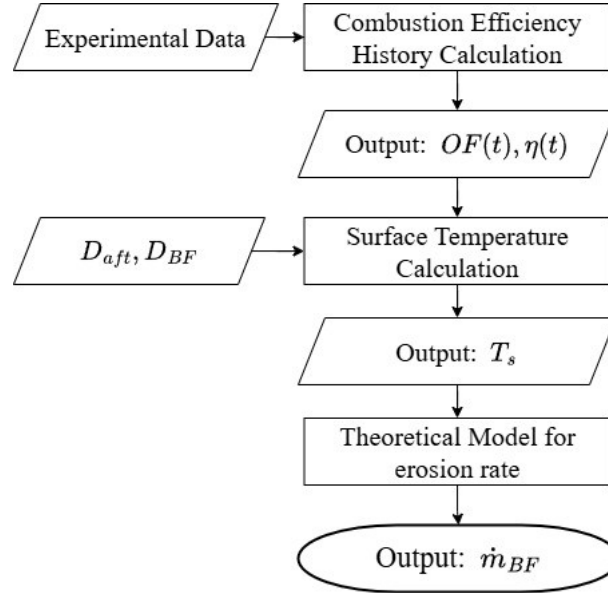


Fig. 5 A calculation flowchart for the erosion rate

1. Combustion efficiency history

This section outlines the methodology employed to determine the efficiency history. The calculation flowchart is shown in Fig. 6. To calculate the efficiency, it is assumed that nozzle erosion is negligible. The inputs for this

calculation include the following: the chamber pressure history, the oxidizer mass flow rate history, the initial fuel geometry, the nozzle throat diameter, and the total fuel consumption. To calculate the regression rate, the formula established by Ito et al.[15] for the HDPE/ N₂O case was used, which is shown in Eq. (8).

$$\dot{r}(t) = K G_{ox}(t)^{0.53} P_C(t)^{0.43} \quad (8)$$

where K is the correction factor to ensure that the experimental mass consumption agrees with the calculated mass consumption based on the fuel mass flow rate expressed as Eq. (9).

$$\dot{m}_f(t) = 2\pi r(t) \dot{r}(t) L_f \rho_f \quad (9)$$

Combustion efficiency is determined in the same way shown in the previous section by using Eqs. (4) and (5).

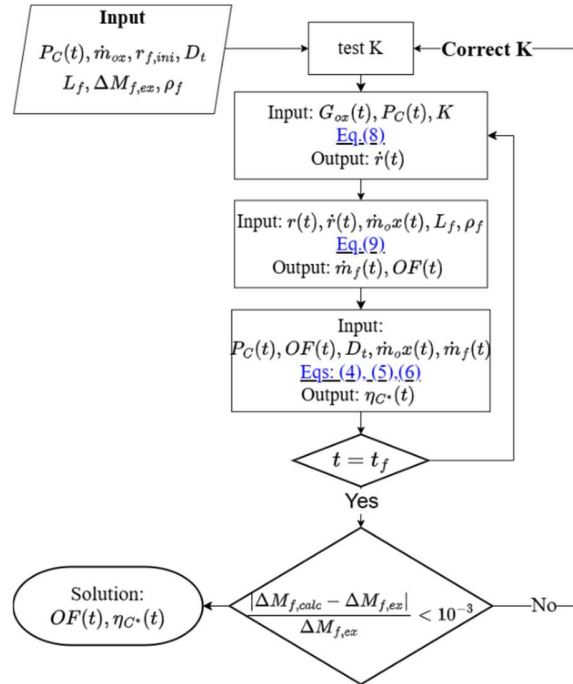


Fig. 6 Calculation flowchart for the efficiency history

2. One-dimensional heat transfer analysis

The surface temperature history was estimated by solving the one-dimensional heat transfer equation. A schematic diagram for the one-dimensional heat transfer analysis is shown in Fig. 7. A mesh was generated along the axial centerline of the baffle plate. In general, the governing equation for one-dimensional conductive heat transfer,

assuming no internal heat generation, is given by Eq. (10). The numerical solution was obtained by taking the first-order upwind difference scheme for time and the second-order central difference scheme for spatial discretization.

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad (10)$$

The boundary conditions assumed convective heat transfer at the upper surface of the baffle plate and the insulation condition at its bottom surface. The formula for the convective heat-transfer coefficient of the impingement flow was calculated using the formula developed by Hrycak [16] as shown in Eq. (11).

$$Nu = \frac{h_g D}{k} = 1.85 C Re^{0.695} Pr^{0.33} \left(\frac{D_{aft}}{D_{BF}} \right)^{1.05} \quad (11)$$

where C is the correction factor that makes the experimental result agree with the calculation result. As a result, C is 0.15.

By solving the heat transfer matrix shown in Eq. (12), the temperature of each position was calculated.

$$\begin{bmatrix} \frac{k}{\Delta x} + h_g & -\frac{k}{\Delta x} & 0 & 0 & \dots & 0 & 0 & 0 \\ a(T_2^t) & b(T_2^t) & a(T_2^t) & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & a(T_{n-1}^t) & b(T_{n-1}^t) & a(T_{n-1}^t) \\ 0 & 0 & 0 & 0 & \dots & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} T_1^{t+\Delta t} \\ T_2^{t+\Delta t} \\ \vdots \\ T_{n-1}^{t+\Delta t} \\ T_n^{t+\Delta t} \end{bmatrix} + \begin{bmatrix} h_g T_g^t \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta x^2 \\ \vdots \\ \Delta x^2 \\ 0 \end{bmatrix} \begin{bmatrix} T_1^t \\ T_2^t \\ \vdots \\ T_{n-1}^t \\ T_n^t \end{bmatrix} = 0 \quad (12)$$

$$a(T) = -\frac{k(T)}{\rho_g C_p(T)} \Delta t \quad (13)$$

$$b(T) = \Delta x^2 + \frac{2k(T)}{\rho C_p(T)} \Delta t$$

where an empirical correlation for thermal conductivity and specific heat are given by $k(T) = 3712T^{-0.6}$ W/(m K) for $200\text{K} < T < 2500\text{ K}$ (Ref. [17]) and $C_p(T) = 651 \ln(T) - 2877$ J/(kg K) for $200\text{ K} < T < 3000\text{ K}$ (Ref. [18]) respectively.

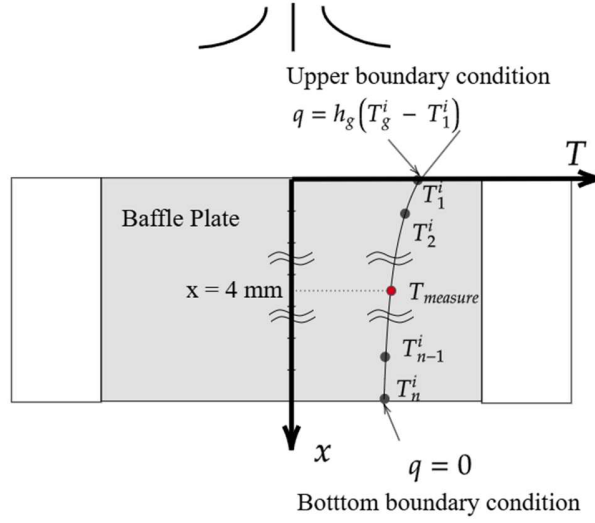


Fig. 7 A schematic diagram for one-dimensional heat transfer analysis

3. Theoretical model for the erosion rate

In order to model the baffle plate erosion phenomenon in an impingement flow configuration, Makino's model, which was originally built for carbon combustion, was used. Figure 8 illustrates a model diagram. At first, the following assumptions are adopted based on Makino's framework: (1) gas-phase reactions are neglected, (2) Prandtl and Schmidt numbers are unity, (3) heat capacity and molecular weight are constant across the boundary layer, and (4) $\rho\mu$ is constant across the boundary layer. Regarding (1) assumption, Rotondi et al. [12] investigated the effect of gas-phase reactions on nozzle erosion through numerical analysis in hybrid rockets. They showed that reactive and frozen simulations provide extremely similar nozzle erosion rate under conditions of low O_2 concentration, because the gas-phase reaction between carbon monoxide and oxygen is negligible; however, under O-rich conditions, the erosion rate can be altered by the exothermic reaction between carbon monoxide and oxygen. To ensure that Assumption 1 holds, all experiments in this study were conducted under fuel-rich or near-stoichiometric conditions.

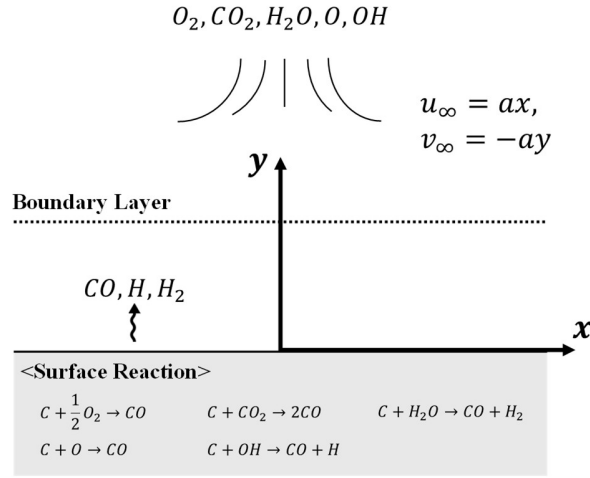


Fig. 8 A schematic diagram of theoretical model for the prediction of erosion rates

The steady-state two-dimensional boundary-layer flow is described by the expressions in Eq. (14). This equation is established not only for O_2 , H_2O and CO_2 as in the original model by Makino but also for O and OH radicals, which have been assumed by many hybrid rocket researchers as the primary oxidizing species in combustion gas[8,12].

$$\rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} - \frac{\partial}{\partial y} \left(\rho D' \frac{\partial Y_i}{\partial y} \right) = 0 \quad (i = O_2, CO_2, H_2O, O, OH) \quad (14)$$

where the subscript i indicates the chemical species.

The velocity component u_∞ and v_∞ of the potential flow outside the boundary layer are given by using the velocity gradient a as shown in Eq. (15).

$$u_\infty = ax, \quad v_\infty = -ay \quad (15)$$

At the carbon surface, the species conservation equations are formulated as Eqs. (16) and (17) by considering the transport of species from the gas phase to the solid surface through diffusion and convection, as well as the consumption due to chemical reactions. As evident from the right-hand side of Eq. (16), all chemical reactions are assumed as the first-order reaction[11].

$$(\rho v Y_i)_s - \left(\rho D' \frac{\partial Y_i}{\partial y} \right) = -(\rho Y_i)_s k_i \quad (16)$$

$$k_i = B_i T_s^{b_i} \exp \left(-\frac{E_i}{RT_s} \right) \quad (17)$$

To calculate the heterogeneous reactions on the carbon surface, the semi-global graphite-oxidation kinetics developed by Chelliah et al. [19] are employed. This research provided two sets of chemical reaction constants: one for non-porous graphite and another for porous graphite. According to this research, the constant set for porous graphite demonstrated better predictive accuracy for experimental results with a graphite density of 1820 kg/m³. In our experiments, the graphite density ranged from 1740 ± 40 kg/m³, indicating the graphite in our experiments was more porous than that of Chelliah et al. [19]. Therefore, in this calculation, the constants for the porous type are used. Table 1 provides a detailed list of these constants.

Table 1 Heterogeneous rate constant of carbon [19]

	B_i	E_i [kJ/mol]	b_i
$C + CO_2 \rightarrow 2CO$	6.00×10^7	266	0.0
$C + H_2O \rightarrow CO + H_2$	6.00×10^7	266	0.0
$C + 1/2 O_2 \rightarrow CO$	2.20×10^6	178	0.0
$C + O \rightarrow CO$	3.41	0.0	0.5
$C + OH \rightarrow CO + H$	1.65	0.0	0.5

Regarding the coordinate system employed in the boundary layer, the variables s and η are defined as:

$$s = \int_0^x \rho_\infty(x) \mu_\infty(x) u_\infty(x) dx \quad (18)$$

$$\eta = \frac{u_\infty(x)}{\sqrt{2s}} \int_0^y \rho(x, y) dy \quad (19)$$

The dimensionless stream function $f(s, \eta)$ is defined based on the stream function $\Psi(s, \eta)$ as follows.

$$f(s, \eta) = \frac{\Psi(s, \eta)}{\sqrt{2s}} \quad (20)$$

$$\rho_{ur} = \frac{\partial \Psi}{\partial y}, \quad \rho_{vr} = -\frac{\partial \Psi}{\partial x} \quad (21)$$

By using the dimensionless parameters, Eq. (16) can be described as:

$$-\left(\frac{d\tilde{Y}_i}{d\eta}\right)_s + (-f_s)\tilde{Y}_{i,s} = A_{s,i}\tilde{Y}_{i,s} \quad (22)$$

where

$$\omega(-f_s) = A_{s,O_2}\tilde{Y}_{O_2,s} + A_{s,CO_2}\tilde{Y}_{CO_2,s} + A_{s,H_2O}\tilde{Y}_{H_2O,s} + A_{s,O}\tilde{Y}_{O,s} + A_{s,OH}\tilde{Y}_{OH,s} \quad (23)$$

$$A_{s,i} = \frac{k_i}{\sqrt{a(\mu_\infty/\rho_\infty)}} \quad (24)$$

where $\tilde{Y}_i$ describes the dimensionless mass fraction defined as $\tilde{Y}_i = (v_{CO_2} MW_{CO_2} / v_i MW_i) Y_i$, $\omega = MW_{CO_2} / MW_C$ is the molecular weight ratio. $A_{i,s}$ is a surface Damköhler number, which is defined as the ratio of the surface chemical reaction rate to the species diffusion rate in the boundary layer and defined in the same way as Makino et al. [11].

The surface mass fraction must be determined to solve Eq. (23). According to Makino[20], by using the convective-diffusive operator defined as $\mathcal{L} = \frac{d^2}{d\eta^2} + f \frac{d}{d\eta}$, the Eq. (14) can be reduced as $\mathcal{L}(\tilde{Y}_i) = 0$. Therefore, the arbitrary mass fraction inside the boundary layer $\tilde{Y}_i$ is described with the profile function ξ as follows:

$$\frac{\tilde{Y}_i - \tilde{Y}_{i,\infty}}{\tilde{Y}_{i,\infty} - \tilde{Y}_{i,s}} = \xi \quad (25)$$

$$\xi = \frac{\int_0^\eta \exp(-\int_0^\eta f d\eta) d\eta}{\int_0^\infty \exp(-\int_0^\eta f d\eta) d\eta} \quad (26)$$

By combining Eq. (22) with Eq. (25) and the transfer number β , which is defined as $\beta \equiv (-f_s) / (d\xi/d\eta)_s$, the surface mass fraction can be described as Eq. (27).

$$\tilde{Y}_{i,s} = \frac{\tilde{Y}_{i,\infty}}{1 + \beta + A_{i,s} \frac{\beta}{(-f_s)}} \quad (27)$$

Similar to the approach taken by many researchers in the field of nozzle erosion[12,21], each mass fraction above the boundary layer was obtained using the NASA CEA code[13]. By employing the relation, $(-f_s) \ll 1$, which is typical for most solid combustion, the following approximate equation can be derived in the same way in the previous research[22].

$$\frac{\beta}{1 + \beta} = 1 - \exp(-K'(-f_s)) \approx K'(-f_s) \quad (28)$$

where K is the correction factor and the approximate expression is defined by the analogy of the heat and mass transfer as Eq. (29).

$$K' = \sqrt{\frac{2}{3} \left(\frac{\tilde{T}_\infty}{\tilde{T}_s} \right) \left(1 - \frac{\tilde{T}_\infty}{2\tilde{T}_s} \right)} + \sqrt{\frac{\pi}{2}} \quad (29)$$

By using Eqs. (23), (27), and (28), the approximation expression for β can be described as follows:

$$\beta = \frac{1}{\omega} \sum \left(\frac{K' A_{i,s}}{1 + K' A_{i,s}} \right) \tilde{Y}_{i,\infty} \quad (i = O_2, CO_2, H_2O, O, OH) \quad (30)$$

Combining Eqs. (28), (29) and (30) yields $(-f_s)$. Finally, the erosion rate is calculated from Eq. (31).

$$\dot{m}_{BF} = (\rho v)_s = (-f_s) \sqrt{a \mu_\infty \rho_\infty} \quad (31)$$

4. Uncertainty analysis

The discrepancies in erosion rate between the theoretical model and the experimental results were estimated via an error-propagation analysis that accounts for both measurement uncertainty and uncertainty in the estimation of input parameters. For the theoretical-model uncertainty analysis, the error sources were taken to be the velocity gradient, gas-species mass fractions, surface temperature, combustion gas temperature, and chamber pressure. The chamber-pressure uncertainty was set to 0.01 MPa based on the pressure-sensor measurement error. For the other four parameters, the uncertainty was assumed to be 5% of the computed values in order to assess the sensitivity of the predicted error to each parameter. The resulting uncertainty in the predicted erosion rate was evaluated from Eq. (32), which is derived from Eq. (31). Because the theoretical model depends on the parameters in a nonlinear manner, the detailed derivation is provided in the Appendix. For the experimental results, the erosion-rate uncertainty was evaluated from Eq. (33) based on Eq. (1). The measurement uncertainty of the erosion depth was set to 0.01 mm, the uncertainty in burn duration to 1 s, and the uncertainty in graphite density to 40 kg/m³, based on the observed scatter in the measured values.

$$\Delta \dot{m}_{BF,theory} = \sqrt{\left(\frac{\partial \dot{m}_{BF}}{\partial f_s} \Delta f_s \right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial a} \Delta a \right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial P_\infty} \Delta P_\infty \right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial T_\infty} \Delta T_\infty \right)^2} \quad (32)$$

$$\Delta \dot{m}_{BF,exp} = \sqrt{\left(\frac{\Delta L_{er}}{t_b} \Delta \rho_g \right)^2 + \left(\frac{\rho_g}{t_b} \Delta(\Delta L_{er}) \right)^2 + \left(\frac{\rho_g \Delta L_{er}}{t_b^2} \Delta t_b \right)^2} \quad (33)$$

III. Experimental and Theoretical Calculation Results

A. Experimental result

A total of 25 experiments were carried out, consisting of three experimental series: the Velocity Gradient Series (VG series) with fifteen tests, the Chamber Pressure Series (Pc series) with six tests, and the Trial Series with four tests. Table 2 summarizes the experimental results. Figure 9 shows the post-test condition of the baffle plate after firing with substantial erosion. No erosion-induced damage or structural failure of the baffle plate was observed.

Table 2 Experimental Result

	t_b [s]	$\bar{P}_{BF}$ [MPa]	$\bar{\dot{m}}_{ox}$ [g/s]	D_t [mm]	$a(\times 10^4)$ [s ⁻¹]	OF [-]	η_{c^*} [-]	ΔL_{er} [mm]	$T_{c,ex}$ [K]
<i>Velocity Gradient series</i>									
MP-D6	14.7	0.883	5.00	3.50	3.03	7.20	0.82	1.07	2169
MP-D6-2	14.9	0.866	5.16	3.49	2.85	7.40	0.78	1.34	1967
MP-D6-3	14.8	0.819	4.97	3.49	2.61	7.65	0.76	0.87	1871
MP-D6-4	14.7	0.810	5.06	3.49	3.23	7.81	0.83	1.16	2207
MP-D6-5	14.8	0.809	5.07	3.54	3.24	7.68	0.85	1.06	2328
MP-D8	14.8	0.881	4.98	3.52	1.77	7.37	0.84	0.80	2277
MP-D8-2	14.9	0.839	4.73	3.48	1.61	6.99	0.79	0.49	1991
MP-D8-3	14.9	0.857	5.07	3.50	1.65	7.88	0.80	0.70	2084
MP-D8-4	14.9	0.867	5.09	3.48	1.59	8.16	0.80	0.57	2086
MP-D12	14.7	0.892	5.17	3.55	0.85	7.46	0.85	0.90	2363
MP-D12-2	14.9	0.859	4.91	3.50	0.78	7.09	0.81	0.75	2132
MP-D12-3	14.8	0.846	4.98	3.48	0.73	7.35	0.78	0.48	1983
MP-D6L8. 5-1	14.9	0.873	5.09	3.52	1.66	7.20	0.78	0.90	1985
MP-D6L8. 5-2	15.0	0.906	5.07	3.48	1.66	7.00	0.81	0.74	2136
MP-D6L8. 5-3	14.8	0.887	4.99	3.50	1.55	7.20	0.81	0.79	2107
<i>Chamber Pressure series</i>									
HP-D6-1	14.8	1.08	5.07	2.99	2.40	7.39	0.80	1.08	2109
HP-D6-2	14.8	1.06	5.02	3.02	2.14	7.47	0.80	1.03	2077
MP-D7-1	14.8	0.832	5.05	3.47	2.30	7.58	0.83	1.00	2218
MP-D7-2	14.8	0.797	4.97	3.50	2.28	7.51	0.82	0.89	2173
LP-D8-1	14.8	0.634	5.01	3.98	2.45	7.58	0.84	0.64	2245
LP-D8-2	14.8	0.643	5.08	3.99	2.43	7.76	0.84	0.78	2273
<i>Trial series</i>									
HP-D7-3	14.9	1.10	5.43	3.03	1.55	8.02	0.75	0.51	1858
LP-D8-3	14.8	0.752	3.94	3.49	1.75	6.78	0.84	0.36	2285
LP-D6-1	14.8	0.634	5.07	3.99	4.06	7.44	0.82	1.16	2172
LP-D6-2	14.8	0.632	4.99	3.98	3.91	7.31	0.82	1.10	2164

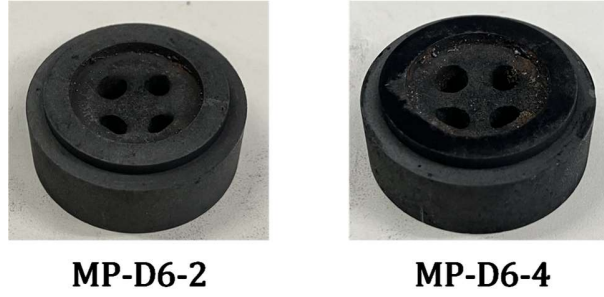


Fig. 9 Post-test conditions of the baffle plate

Figure 10 illustrates the pressure histories for the VG Series, showing that the pressure in each experiment remained stable within a range from 0.7 to 0.83 MPa. Conversely, in the Pc Series, the pressure upstream of the baffle plate was varied from 0.65 to 1.08 MPa as a parameter, while the velocity gradient was kept constant at $2.30 \times 10^4 \pm 0.15 \times 10^4 \text{ s}^{-1}$. As shown in Fig. 10, the pressure remained stable throughout the firing duration, indicating the absence of nozzle erosion and thereby supporting the assumption used to calculate the combustion efficiency history in Section II .B.1.

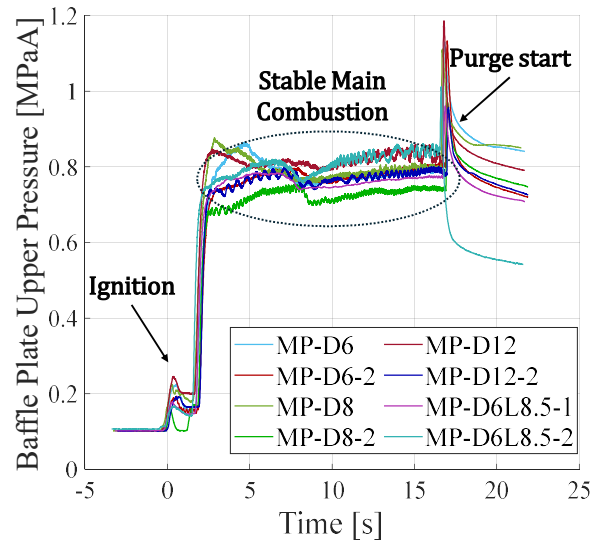


Fig. 10 Baffle Plate upper pressure history of VG series. The chamber pressure in each test remained stable within the range of 0.70–0.83 MPa throughout the main combustion

Figure 11 illustrates the erosion rate in the VG series as a function of the velocity gradient. The dotted line indicates the trend line of the erosion rate, showing the positive correlation between the erosion rate and the velocity

gradient. The least squares method gives the velocity gradient exponent of 0.384. This value is smaller than 0.5 reported by Day et al.[10], even though the velocity gradient is in the same range: from 2900 to 40000 s^{-1} . Moreover, Fig. 11 reveals minor scatter at the same velocity gradient even though the pressure history and O/F ratio are almost the same in the VG series experiments, as shown in Fig. 10 and Table 2. The relationship between gas pressure and erosion rates is illustrated in Fig. 12. The pressure exponent, derived from the fitting results, is 0.788. The dependency of each parameter is further analyzed using the theoretical model in a later section.

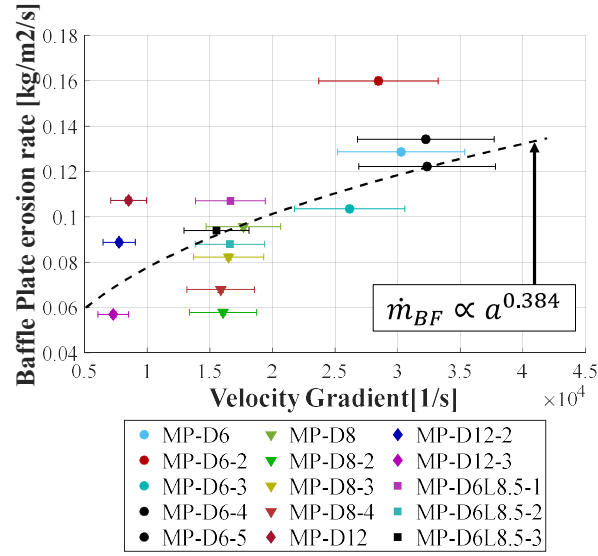


Fig. 11 Velocity gradient exponent for the erosion rate in the Velocity Gradient series experiment. The dotted line shows the least-squares fit, yielding a velocity-gradient exponent of 0.384.

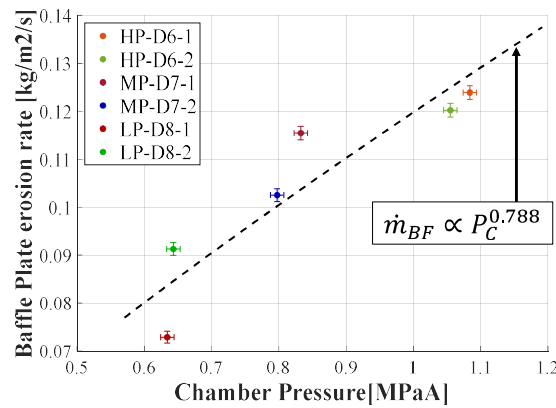


Fig. 12 Relationship between chamber pressure and erosion rates. The dotted line shows the least-squares fit, yielding a pressure exponent of 0.788.

B. Temperature analysis result

The calculated combustion efficiency profile is presented in Fig. 13. This figure shows that efficiency increases drastically from zero to one second and then becomes stable over the entire combustion duration. These trends are similar to the results of the NTRT+ analysis adapted to HDPE/N₂O hybrid rocket [23]. The correction factor K ranges from 1.3 to 1.9. This indicates that the regression rate is higher than that reported by Ito et al. [15]. This difference may stem from the differences in the oxidizer mass flux range. In Ito's experiments, the oxidizer mass flux ranged from 45 to 245 kg/m²s, whereas in our experiments, it ranged from 20 to 100 kg/m²s. The regression rate tends to increase in the low-mass-flux regime due to the radiation[4]. Therefore, it is reasonable for the correction factor to be greater than one.

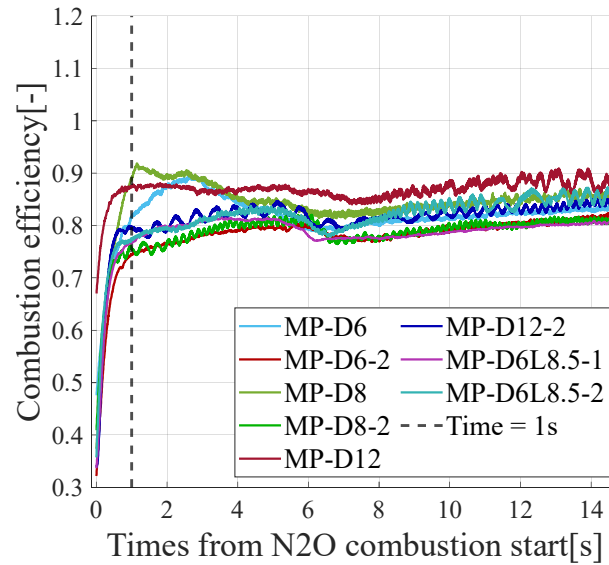
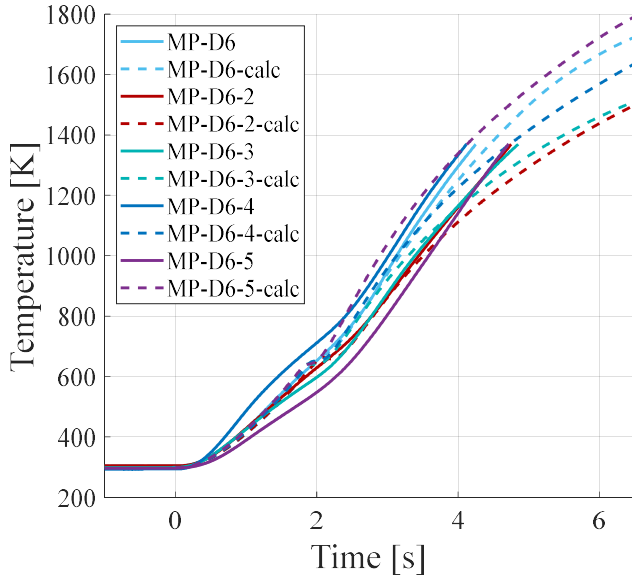
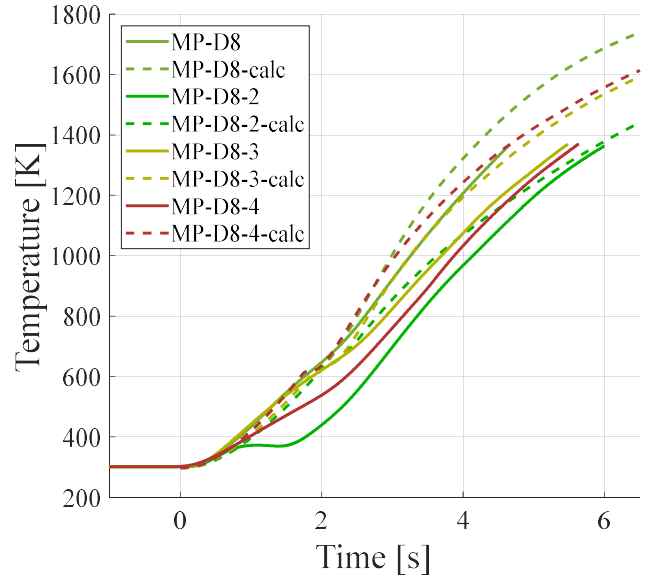


Fig. 13 Combustion efficiency history

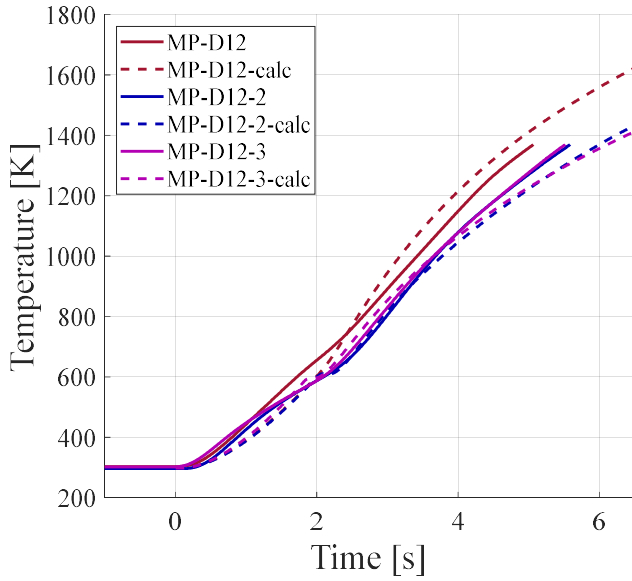
Figure 14 shows the comparison between the calculated temperature history and the measured data at the 4.0 mm below the upper surface of the baffle plate. Since K-type thermocouples are limited to temperatures up to 1370 K, the measurement data exists only up to approximately six seconds. Therefore, the comparison was made within the range where measured temperature data exist. As shown in Fig. 14, the calculation result shows good agreement with the measured temperature. As illustrated in Fig. 14 (b) and (c), the order of time to reach the upper temperature limit agrees with the calculation results: No. 1, No. 3, No. 4, and No. 2 in D8L5 series and No. 1, No. 2 and No. 3 in D12L5 series. Therefore, it is concluded that the heat-transfer calculation adequately predicts the temperature history.



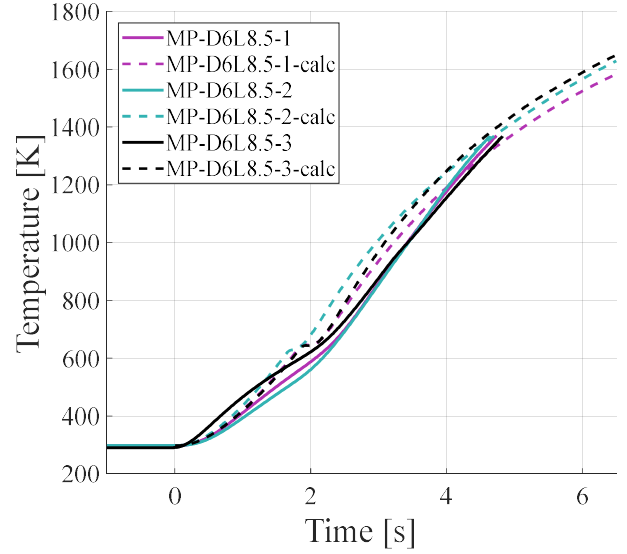
(a) D6 series



(b) D8 series



(c) D12series



(d) D6L8.5 series

Fig. 14 Comparison of the calculated and experimentally measured temperature history

Figure 15 illustrates the surface temperature history. Around the two-second mark, when the oxidizer shifts from oxygen to nitrous oxide, a decrease in surface temperature can be observed. This temperature drop likely reflects the incomplete development of the flame after the supply of nitrous oxide. The incomplete combustion results in the unburned nitrous oxide impinging on the surface. Indeed, as seen in Fig. 14, the temperature rise slowed down after

two seconds in all experiments. These findings suggest that the calculated temperature at the surface accurately reflects the incomplete combustion. Unlike the combustion efficiency history, the steady state seems to be achieved eight seconds after the ignition. Therefore, the steady-state surface temperature was calculated by averaging the temperature history from eight seconds after ignition until extinction. These values are used for the theoretical model, which is shown later.

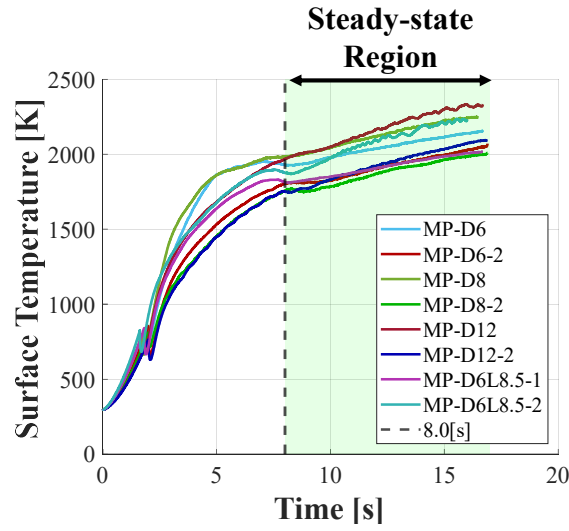


Fig. 15 The surface temperature history. The steady-state temperature is evaluated from eight seconds.

C. Results of Theoretical Model Calculations

Figure 16 illustrates a comparison between the predicted and experimental erosion rates. Although no correlation factor was used to adapt the Makino's theoretical model to the hybrid rocket environments, the coefficient of determination is 0.735 based on 25 experimental data with an average error of 33 %. Almost all experimental erosion rates, except for MP-D12-3, and LP-D8-3, fall within 42% of the predicted erosion rate without any corrections. The uncertainties associated with the experiments are shown as error bars in Fig. 16. The resulting uncertainty was 1.32% on average and at most 2.3% relative to the measured values.

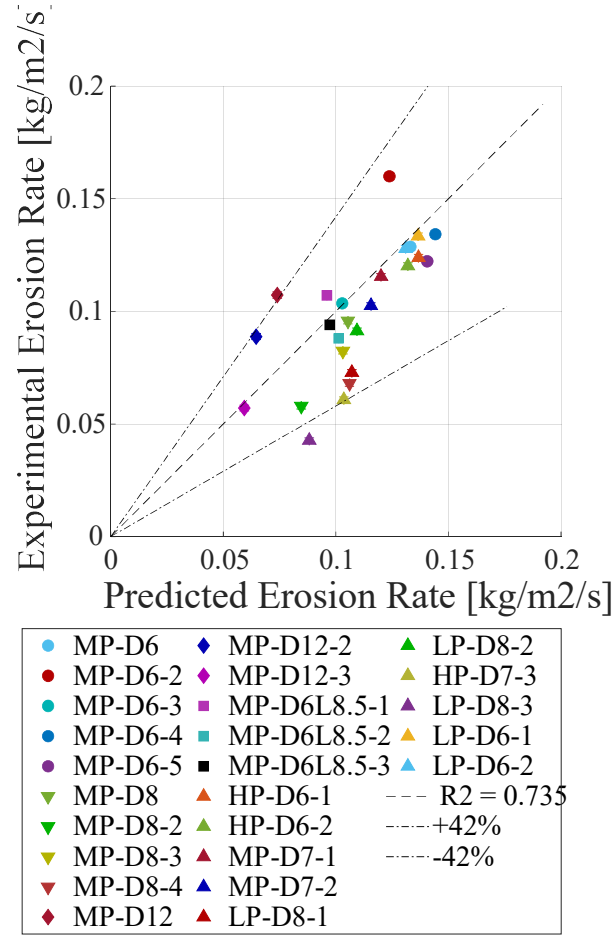


Fig. 16 The comparison with the predicted erosion rate and experimental erosion rate

The uncertainty of the predictive model is discussed here. As described in Sec. II.B.4, the model takes the velocity gradient, surface temperature, chamber pressure, gas temperature, and gas-species mass fractions as the primary inputs. To quantify how much each input's prediction or measurement uncertainty contributes to the overall model error, the uncertainties of four of the five parameters were set to zero and calculated the resulting fractional error in the predicted erosion rate due solely to the remaining parameter. The results are summarized in Table 3. The calculation was performed at a chamber pressure of 1.0 MPa, a velocity gradient of $1.0 \times 10^4 \text{ s}^{-1}$, a surface temperature of 2000 K, and under stoichiometric conditions. Among the inputs, the surface temperature has the largest influence, producing a 36.6% error in the predicted erosion rate. Given that the mean discrepancy between the model predictions and the experimental data was 33%, this suggests that the dominant contributor is the uncertainty in the surface temperature prediction.

Table 3 Primary Sources of Error in the Predictive Model

	Δx	$\left(\frac{\partial \dot{m}_{BF}}{\partial x} \Delta x \right)$
	$[-]$	$\dot{m}_{BF}$ [%]
Velocity Gradient	500 s	2.77
Chamber Pressure	0.01 MPa	0.54
Combustion Gas Temperature	110 K	1.04
Surface Temperature	100 K	36.6
Mass fraction	5% each species	3.00
(sum)		43.9

IV. Discussion on parameter sensitivity**A. Theoretical exponents under diffusion control and comparison with experiments**

The observed scatter in the data and the low velocity-gradient exponent are mainly attributed to the temperature dependence of the erosion rate. Figure 17 illustrates the relationship between the gas temperature and the erosion rate under similar chamber pressure and velocity gradient conditions. The gas temperature was calculated from Eq. (7), accounting for the combustion efficiency. Although the experimental conditions in VG series were carefully controlled to maintain the same O/F ratio and chamber pressure, variations in combustion efficiency led to discrepancies in gas temperature. As shown in Fig. 17, the erosion rate increases with an increase in the gas temperature. This relationship indicates that erosion phenomena are not governed solely by diffusion. As mentioned in the Introduction, the erosion phenomena at the baffle-plate surface are assumed to be diffusion-limited, owing to the high temperature and heat transfer from the combustion gas to the baffle plate, as well as the findings of Walker et al.[9]. However, the observed correlation between temperature and erosion rates indicates that, contrary to expectations, the erosion phenomena are influenced not only by the diffusion but also by the chemical reactions in the temperature range around 2000 K in hybrid rockets, which has also been observed in the field of the nozzle erosion[12,21,24] .

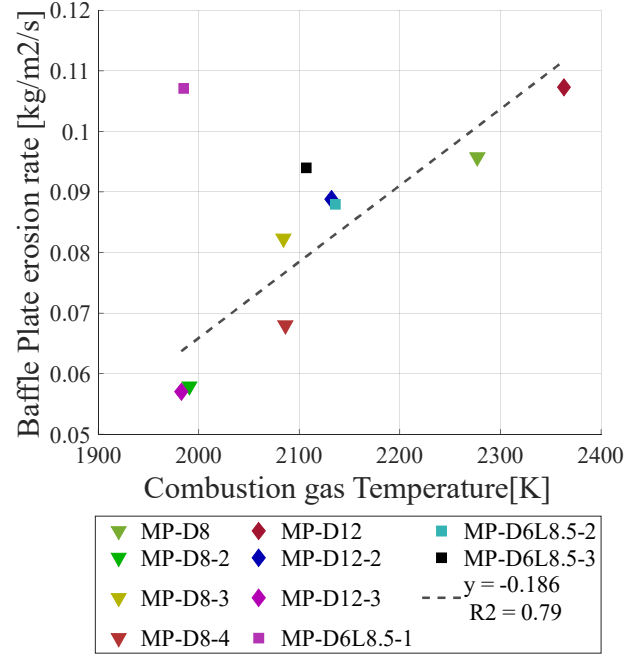


Fig. 17 Relationship between a gas temperature and erosion rate

To discuss whether the erosion phenomena in the experiments were governed by diffusion, the theoretical exponents of the velocity gradient and temperature under diffusion-limited conditions were also analyzed. According to Eq.(16), the erosion rates can be expressed using Eq. (34) in a fully diffusion-limited case. The concentration gradient is assumed to be the ratio between the concentrations in the core flow Y_∞ and the boundary layer thickness δ .

$$\dot{m}_{BF} \approx \rho D' \frac{\partial Y}{\partial x} \approx \rho D' \frac{Y_\infty}{\delta} \quad (34)$$

The boundary layer thickness in impingement flow, assuming that a two-dimensional potential flow is applicable, is as follows[25].

$$\delta = 2.8 \sqrt{\frac{\mu}{\rho a}} \quad (35)$$

When considering temperature dependency, the relationship between diffusion coefficient and temperature must be considered. The following equation gives the diffusion coefficient between two gas phase chemical species[26].

$$D' = \frac{AT^{1.75}}{\left(V_A^{\frac{1}{3}} + V_B^{\frac{1}{3}}\right)^{1.286}} \left[\frac{1}{W_A} + \frac{1}{W_B}\right]^{0.5} \approx T^{1.75}P^{-0.953} \quad (36)$$

where V is the volume of gas, which is assumed as $V \propto P^{-1}$.

Based on Sutherland equation and the perfect gas assumption, it can be assumed that $\mu \approx T^{0.5}$ and $\rho \approx P^1T^{-1}$. These relationships yield the erosion rate expressed as Eq. (37).

$$\dot{m}_{BF} \approx \rho D' \frac{Y_\infty}{\delta} \approx (P^1T^{-1})(P^{-0.953}T^{1.75}) \frac{Y}{P^{-0.5}T^{0.75}a^{-0.5}} \approx P^{0.547}T^0a^{0.5}Y \quad (37)$$

Accordingly, the erosion rate is independent of the temperature and proportional to the 0.5th power of the velocity gradient. Kuchta et al.[27] also derived these relationships. As clearly shown in Eq. (37), the erosion rate should be independent of temperature in the diffusion-limited conditions whereas experimental results indicate that the erosion rate depends on temperature in Fig. 17. Moreover, the exponents obtained from the experiments were 0.38 for the velocity gradient and 0.788 for the chamber pressure, neither of which agreed with the theoretical values expected for diffusion-limited erosion. These findings suggest that the erosion phenomenon was in the regime influenced by both diffusion and chemical reaction rates.

B. Model-based parametric exponents of velocity gradient and pressure

As previously stated in Section IV.A, it is challenging to directly evaluate the effects of velocity gradient and chamber pressure on the erosion rate due to the influence of surface temperature. In this section, the dependency of two parameters at the same surface temperature are discussed based on the theoretical model. The calculation was performed under the following nominal conditions: the equivalence ratio equals one, the chamber pressure is 1.0 MPa, and the velocity gradient is $1.0 \times 10^4 \text{ s}^{-1}$. The dependency of each parameter is depicted in Fig. 18. Regarding the influence of the velocity gradient, the exponent increases toward 0.5 with the surface temperature. Considering the transition to diffusion-limited behavior as the surface temperature rises, it is reasonable for the exponents to converge toward the value discussed in Section IV.A. Similarly, for the effect of pressure, the exponent decreases toward 0.5 as the surface temperature rises. The pressure sensitivity also converges to the value discussed in the previous section, affirming the validity of the arguments presented earlier. However, even at a surface temperature of 1700 K, the exponent remains 0.60, which is lower than the experimentally observed value of 0.77. This discrepancy may arise

because the experimental exponent reflects not only the effect of pressure but also the influence of surface temperature due to the enhanced heat transfer coefficient.

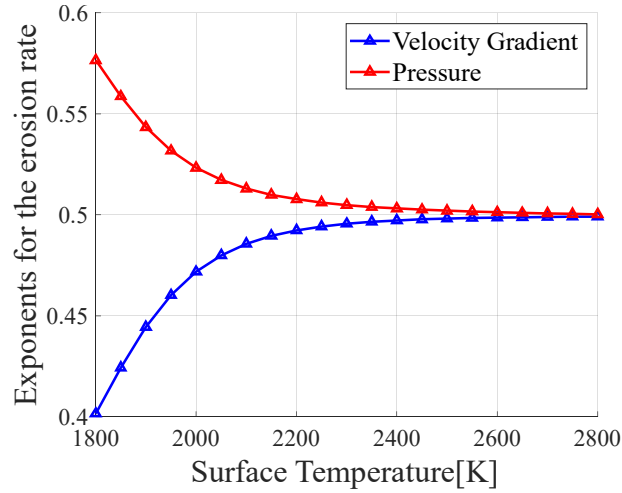


Fig. 18 The relationship between the surface temperature and the exponents for the erosion rate

C. Gas-Composition Effects on the Diffusion-Limited Transition

Unlike previous studies, the erosion phenomenon in the present experiments did not become diffusion-limited. This is because the temperature dependence of the erosion rate is influenced by the gas composition. Table 4 summarizes previous research on the relationship between velocity gradient and carbon combustion rates. According to Day's study, when oxygen is the dominant species of impingement gas, the exponent of the velocity gradient became 0.5, which is equal to the theoretical value under diffusion-limited conditions[10]. On the other hand, as the proportion of H_2O increases, the exponent decreases even within the same surface temperature range. Graham's research clearly shows the H_2O effect on the exponent[28]. Using combustion gas containing approximately 50 % H_2O as the working fluid, the exponents ranged from 0.23 to 0.36. Despite higher surface temperatures compared to those of Day et al., the data suggest that the phenomena were not diffusion-limited. Similar effects of H_2O have also been reported in the studies by Parker[29] and Mayers[30]. This finding implies that increasing H_2O content raises the onset temperature for diffusion-limited conditions.

Table 4 The relationship between the velocity gradient dependency and the gas composition

	Exponents of the velocity gradient [-]	Fluid [-]	Surface temperature K
Day et al.[10]	0. 5	Oxygen (37 ~ 100%)	1500 ~2000
Kuchta et al.[27]	0. 45	Dry air	1338 ~ 1588
Parker et al. [29]	0. 37	Humid air	951 ~ 1422
Mayers [30]	0. 4	Dry air and humid air	1173 ~ 1273
Graham et al. [28]	0. 23 ~ 0. 36	Combustion gas (H_2O 50%)	2533 ~ 2623

The surface Damköhler number of each chemical species provides an explanation for this tendency. Figure 19 illustrates the relationship between surface temperature and surface Damköhler number for O_2 and H_2O . The Damköhler number is calculated using Eq. (24) under the conditions of a chamber pressure of 1.0 MPa and an equivalence ratio of unity. The dotted line represents the point at which the chemical reaction rate equals the diffusion rate. Above this line, the chemical reaction rate surpasses the diffusion rate. As shown in Fig. 19, the onset temperature at which the surface Damköhler number reaches unity is approximately 1550 K for O_2 and 1800 K for H_2O . This difference arises because the activation energy of H_2O is approximately 1.5 times higher than that of O_2 , as shown in Table 1. To reach a diffusion-limited condition, the C- H_2O reaction requires a higher temperature compared to the C- O_2 reaction. Therefore, it is concluded that the experimental results did not exhibit diffusion-limited conditions, since the primary oxidizer in the combustion gas was H_2O .

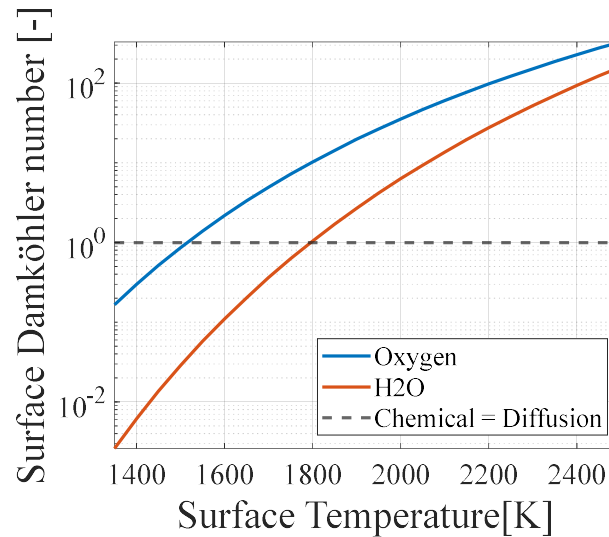


Fig. 19 Relationship between a surface temperature and surface Damköhler number

Because the onset condition for erosion depends on the combustion-gas composition, the fuel–oxidizer combination and the O/F ratio may also serve as governing parameters for the erosion process. Figure 20 shows the gas-species mass fractions as a function of equivalence ratio for representative propellant combinations used in hybrid rockets. When HDPE is used as the fuel and only the oxidizer is varied among H_2O_2 , O_2 , and N_2O , the H_2O fraction for the H_2O_2 case is more than twice that of the other two cases. This observation suggests that the erosion-onset temperature differs for HDPE/ H_2O_2 case. In the O_2 and N_2O cases, under oxidizer-rich operation (equivalence ratio = 0.5), O_2 exhibits the largest mass fraction; as the mixture shifts from stoichiometric toward fuel-rich conditions, the oxygen content decreases while the H_2O fraction increases. When N_2O is used with different fuels (HDPE versus HTPB), no substantial differences in the combustion-gas composition are observed. This suggests that, as long as hydrocarbon fuels are employed, the gas composition does not change markedly. Overall, these results indicate that, in addition to the chamber pressure, velocity gradient, and wall temperature considered thus far, the equivalence ratio and the choice of oxidizer may also need to be taken into account when discussing erosion phenomena.

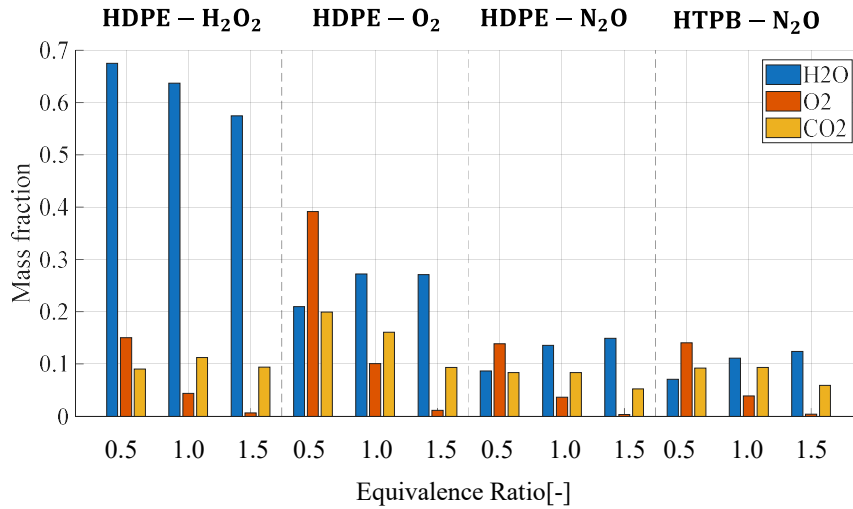


Fig. 20 Relationship Between Equivalence Ratio, Propellant Combination, and Gas Composition

V. Conclusion

In this work, the erosion phenomena at the stagnation point in impingement flow were investigated through experimental and numerical analysis of an HDPE/ N_2O hybrid rocket motor employing a graphitic baffle plate device. The sensitivity of the erosion rate to flow velocity gradient and combustion chamber pressure was investigated in detail, revealing values of 0.38 and 0.77, respectively. The dependency of the velocity gradient was lower than the 0.5 expected under diffusion-limited conditions. A parameter sensitivity analysis using the model revealed that the

pressure exponent sensitivity converges to 0.5 for a purely diffusion-limited process. These results suggest that, contrary to the initial hypothesis, the erosion phenomenon does not exhibit diffusion-limited behavior, despite the high combustion gas temperatures and heat transfer coefficients. This conclusion is further supported by the observed correlation between gas temperature and erosion rate. A comparison with previous studies suggests that the temperature range at which diffusion-limited behavior occurs may vary depending on the dominant oxidizing species in the gas mixture.

By modifying the chemical reaction model in the erosion prediction model established in the field of carbon combustion, the prediction model can capture the erosion rate in the transition region between diffusion-limited and kinetic-limited correctly. The model showed relatively good agreement with experimental results, with a coefficient of determination of 0.735 and an average error of 33% for the erosion rate without any corrections.

Due to nozzle erosion at high-pressure conditions, the current experimental setup imposes an upper chamber pressure limit of approximately 1.2 MPa. Future research should investigate the erosion phenomena at higher pressures to enable a more comprehensive discussion on the pressure effects and the applicability range of the prediction model in high-pressure environments. Additionally, as also indicated by Rotondi et al. [12], under oxidizer-rich operating conditions, the gas-phase volumetric exothermic reaction between carbon monoxide—the primary ablation product of graphite—and oxygen present in the boundary layer can become non-negligible. Consequently, the assumption of negligible gas-phase reactions adopted in the theoretical model may no longer hold. Future work will include combustion tests under oxidizer-rich conditions to investigate the applicability range of the proposed prediction model.

Appendix: Uncertainty analysis equations

$$\Delta \dot{m}_{BF,theo} = \sqrt{\left(\frac{\partial \dot{m}_{BF}}{\partial f_s} \Delta f_s\right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial a} \Delta a\right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial P_\infty} \Delta P_\infty\right)^2 + \left(\frac{\partial \dot{m}_{BF}}{\partial T_\infty} \Delta T_\infty\right)^2}$$

$$\left[\begin{array}{l} \frac{\partial \dot{m}_{BF}}{\partial f_s} = \sqrt{a \mu_\infty \rho_\infty} \\ \frac{\partial \dot{m}_{BF}}{\partial a} = \frac{\sqrt{\frac{\mu_\infty \rho_\infty}{a}} (-f_s)}{2} \\ \frac{\partial \dot{m}_{BF}}{\partial P_\infty} = \frac{\sqrt{a \mu_\infty \rho_\infty} (-f_s)}{2 P_\infty} \\ \frac{\partial \dot{m}_{BF}}{\partial T_\infty} = \frac{\sqrt{a \mu_\infty \rho_\infty} (-f_s)}{2 T_\infty^2} \end{array} \right] \quad (38)$$

$$\Delta(-f_s) = \sqrt{\left(\frac{\partial(-f_s)}{\partial K'} \Delta K'\right)^2 + \left(\frac{\partial(-f_s)}{\partial \beta} \Delta \beta\right)^2}$$

$$\left[\begin{array}{l} \frac{\partial(-f_s)}{\partial K'} = -\frac{1}{K'^2} \frac{\beta}{1+\beta} \\ \frac{\partial(-f_s)}{\partial \beta} = \frac{1}{K'} \frac{1}{(1+\beta)^2} \end{array} \right] \quad (39)$$

$$\Delta K' = \sqrt{\left(\frac{\partial K'}{\partial \tilde{T}_\infty} \Delta \tilde{T}_\infty\right)^2 + \left(\frac{\partial K'}{\partial \tilde{T}_s} \Delta \tilde{T}_s\right)^2}$$

$$\left[\begin{array}{l} \frac{\partial K'}{\partial \tilde{T}_\infty} = \frac{\tilde{T}_s - \tilde{T}_\infty}{\tilde{T}_s^2 \sqrt{6 \left(\frac{\tilde{T}_\infty}{\tilde{T}_s}\right) \left(1 - \frac{\tilde{T}_\infty}{2\tilde{T}_s}\right)}} \\ \frac{\partial K'}{\partial \tilde{T}_s} = \frac{\tilde{T}_\infty^2 - \tilde{T}_s \tilde{T}_\infty}{\tilde{T}_s^3 \sqrt{6 \left(\frac{\tilde{T}_\infty}{\tilde{T}_s}\right) \left(1 - \frac{\tilde{T}_\infty}{2\tilde{T}_s}\right)}} \end{array} \right] \quad (40)$$

$$\Delta \beta = \sqrt{\left(\frac{\partial \beta}{\partial \tilde{Y}} \Delta \tilde{Y}\right)^2 + \left(\frac{\partial \beta}{\partial K'} \Delta K'\right)^2 + \left(\frac{\partial \beta}{\partial A_{i,s}} \Delta A_{i,s}\right)^2}$$

$$\left[\begin{array}{l} \frac{\partial \beta}{\partial \tilde{Y}} = \frac{1}{\omega} \Sigma \left(\frac{K' A_{i,s}}{1 + K' A_{i,s}} \right) \\ \frac{\partial \beta}{\partial K'} = \frac{1}{\omega} \Sigma \left(\frac{A_{i,s}}{(1 + K' A_{i,s})^2} \right) \tilde{Y}_{i,\infty} \\ \frac{\partial \beta}{\partial A_{i,s}} = \frac{1}{\omega} \Sigma \left(\frac{K'}{(1 + K' A_{i,s})^2} \right) \tilde{Y}_{i,\infty} \end{array} \right] \quad (41)$$

$$\Delta A_{i,s} = \sqrt{\left(\frac{\partial A_{i,s}}{\partial a} \Delta a\right)^2 + \left(\frac{\partial A_{i,s}}{\partial P_\infty} \Delta P_\infty\right)^2 + \left(\frac{\partial A_{i,s}}{\partial T_\infty} \Delta T_\infty\right)^2 + \left(\frac{\partial A_{i,s}}{\partial T_s} \Delta T_s\right)^2}$$

$$\left[\begin{array}{l} \frac{\partial A_{i,s}}{\partial a} = A_{i,s} \left(-\frac{1}{2}\right) a^{-\frac{1}{2}} \\ \frac{\partial A_{i,s}}{\partial P_\infty} = A_{i,s} \left(\frac{1}{2}\right) P_\infty^{-1} \\ \frac{\partial A_{i,s}}{\partial T_\infty} = A_{i,s} \left(-\frac{1}{2}\right) T_\infty^{-2} \\ \frac{\partial A_{i,s}}{\partial a} = A_{i,s} \frac{E_i}{RT_s^2} \end{array} \right] \quad (42)$$

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